6 | PHOTONS AND MATTER WAVES

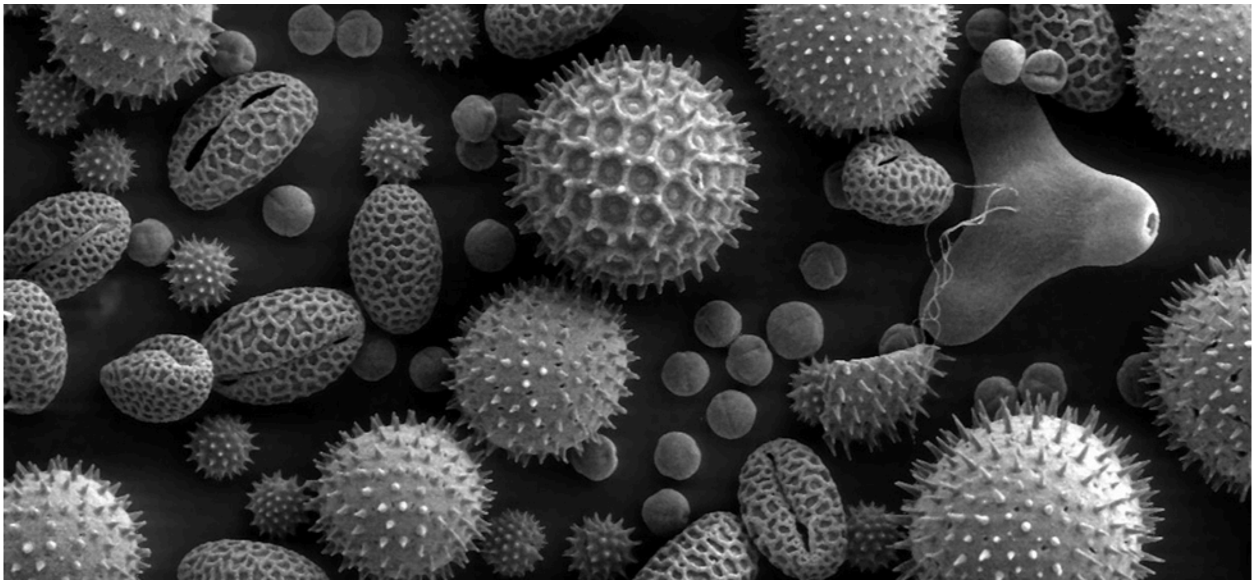


Figure 6.1 In this image of pollen taken with an electron microscope, the bean-shaped grains are about $50\mu\text{m}$ long. Electron microscopes can have a much higher resolving power than a conventional light microscope because electron wavelengths can be 100,000 times shorter than the wavelengths of visible-light photons. (credit: modification of work by Dartmouth College Electron Microscope Facility)

Chapter Outline

Introduction

Two of the most revolutionary concepts of the twentieth century were the description of light as a collection of particles, and the treatment of particles as waves. These wave properties of matter have led to the discovery of technologies such as electron microscopy, which allows us to examine submicroscopic objects such as grains of pollen, as shown above.

In this chapter, you will learn about the energy quantum, a concept that was introduced in 1900 by the German physicist Max Planck to explain blackbody radiation. We discuss how Albert Einstein extended Planck’s concept to a quantum of light (a “photon”) to explain the photoelectric effect. We also show how American physicist Arthur H. Compton used the photon concept in 1923 to explain wavelength shifts observed in X-rays. After a discussion of Bohr’s model of hydrogen, we describe how matter waves were postulated in 1924 by Louis-Victor de Broglie to justify Bohr’s model and we examine the experiments conducted in 1923–1927 by Clinton Davisson and Lester Germer that confirmed the existence of de Broglie’s matter waves.

6.1 | Blackbody Radiation

Learning Objectives

By the end of this section you will be able to:

- Apply Wien’s and Stefan’s laws to analyze radiation emitted by a blackbody
- Explain Planck’s hypothesis of energy quanta

All bodies emit electromagnetic radiation over a range of wavelengths. In an earlier chapter, we learned that a cooler body radiates less energy than a warmer body. We also know by observation that when a body is heated and its temperature rises, the perceived wavelength of its emitted radiation changes from infrared to red, and then from red to orange, and so forth. As its temperature rises, the body glows with the colors corresponding to ever-smaller wavelengths of the electromagnetic spectrum. This is the underlying principle of the incandescent light bulb: A hot metal filament glows red, and when heating continues, its glow eventually covers the entire visible portion of the electromagnetic spectrum. The temperature (T) of the object that emits radiation, or the **emitter**, determines the wavelength at which the radiated energy is at its maximum. For example, the Sun, whose surface temperature is in the range between 5000 K and 6000 K, radiates most strongly in a range of wavelengths about 560 nm in the visible part of the electromagnetic spectrum. Your body, when at its normal temperature of about 300 K, radiates most strongly in the infrared part of the spectrum.

Radiation that is incident on an object is partially absorbed and partially reflected. At thermodynamic equilibrium, the rate at which an object absorbs radiation is the same as the rate at which it emits it. Therefore, a good **absorber** of radiation (any object that absorbs radiation) is also a good emitter. A perfect absorber absorbs all electromagnetic radiation incident on it; such an object is called a **blackbody**.

Although the blackbody is an idealization, because no physical object absorbs 100% of incident radiation, we can construct a close realization of a blackbody in the form of a small hole in the wall of a sealed enclosure known as a cavity radiator, as shown in **Figure 6.2**. The inside walls of a cavity radiator are rough and blackened so that any radiation that enters through a tiny hole in the cavity wall becomes trapped inside the cavity. At thermodynamic equilibrium (at temperature T), the cavity walls absorb exactly as much radiation as they emit. Furthermore, inside the cavity, the radiation entering the hole is balanced by the radiation leaving it. The emission spectrum of a blackbody can be obtained by analyzing the light radiating from the hole. Electromagnetic waves emitted by a blackbody are called **blackbody radiation**.

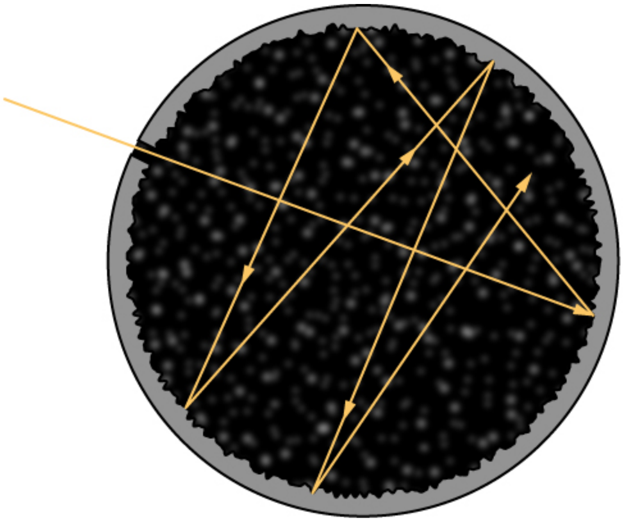


Figure 6.2 A blackbody is physically realized by a small hole in the wall of a cavity radiator.

The intensity $I(\lambda, T)$ of blackbody radiation depends on the wavelength λ of the emitted radiation and on the temperature T of the blackbody (**Figure 6.3**). The function $I(\lambda, T)$ is the **power intensity** that is radiated per unit wavelength; in other words, it is the power radiated per unit area of the hole in a cavity radiator per unit wavelength. According to this definition, $I(\lambda, T)d\lambda$ is the power per unit area that is emitted in the wavelength interval from λ to $\lambda + d\lambda.$ The intensity distribution among wavelengths of radiation emitted by cavities was studied experimentally at the end of the nineteenth century. Generally, radiation emitted by materials only approximately follows the blackbody radiation curve (**Figure 6.4**); however, spectra of common stars do follow the blackbody radiation curve very closely.

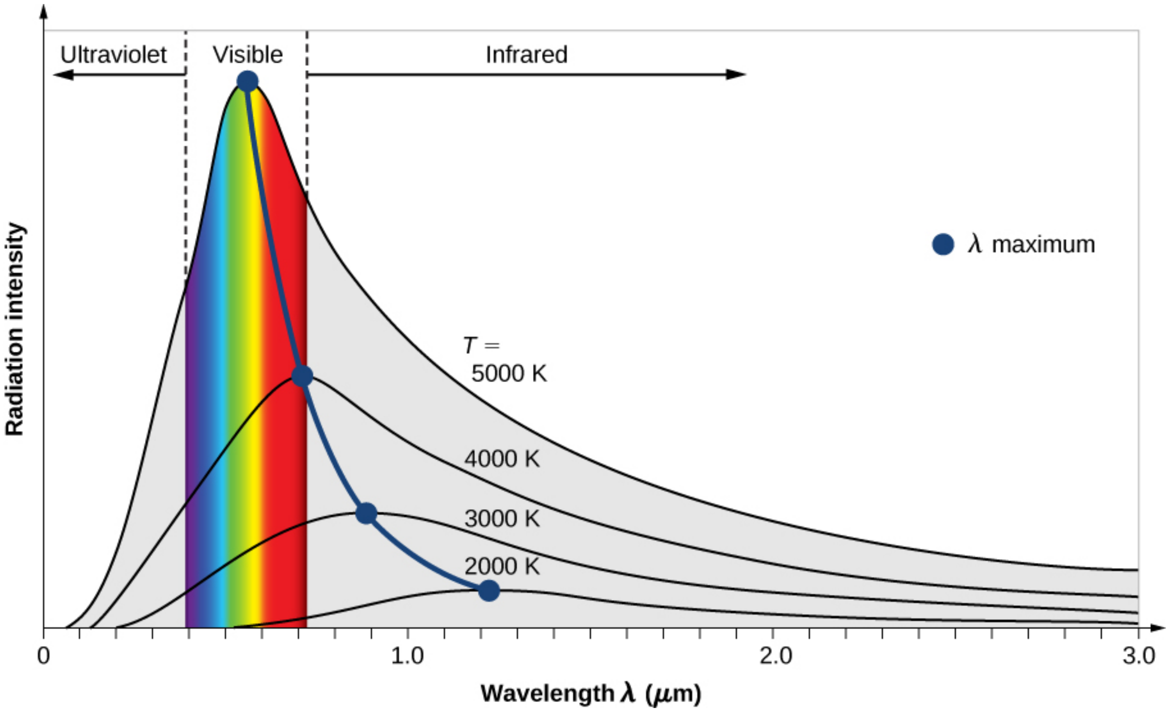


Figure 6.3 The intensity of blackbody radiation versus the wavelength of the emitted radiation. Each curve corresponds to a different blackbody temperature, starting with a low temperature (the lowest curve) to a high temperature (the highest curve).

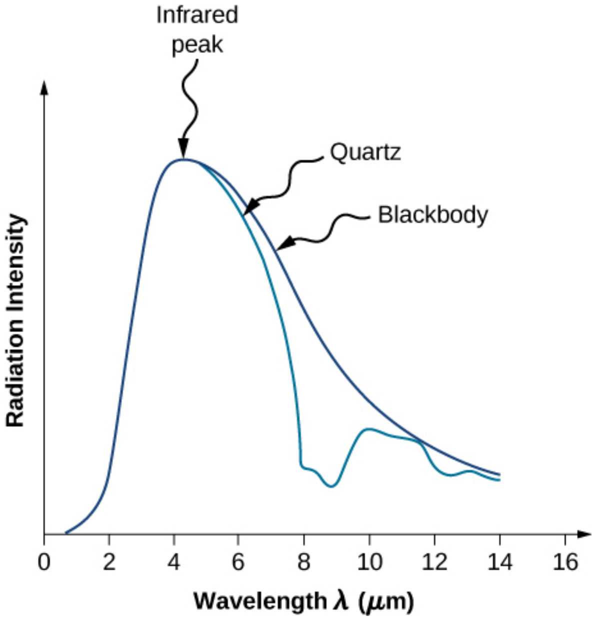


Figure 6.4 The spectrum of radiation emitted from a quartz surface (blue curve) and the blackbody radiation curve (black curve) at 600 K.

Two important laws summarize the experimental findings of blackbody radiation: *Wien's displacement law* and *Stefan's law*. Wien's displacement law is illustrated in **Figure 6.3** by the curve connecting the maxima on the intensity curves. In these curves, we see that the hotter the body, the shorter the wavelength corresponding to the emission peak in the radiation curve. Quantitatively, Wien's law reads

$$\lambda_{\text{max}} T = 2.898 \times 10^{-3}\,\text{m} \cdot \text{K} \tag{6.1}$$

where λ_{max} is the position of the maximum in the radiation curve. In other words, λ_{max} is the wavelength at which a blackbody radiates most strongly at a given temperature T. Note that in **Equation 6.1**, the temperature is in kelvins. Wien's displacement law allows us to estimate the temperatures of distant stars by measuring the wavelength of radiation they emit.

Example 6.1

Temperatures of Distant Stars

On a clear evening during the winter months, if you happen to be in the Northern Hemisphere and look up at the sky, you can see the constellation Orion (The Hunter). One star in this constellation, Rigel, flickers in a blue color and another star, Betelgeuse, has a reddish color, as shown in **Figure 6.5**. Which of these two stars is cooler, Betelgeuse or Rigel?

Strategy

We treat each star as a blackbody. Then according to Wien's law, its temperature is inversely proportional to the wavelength of its peak intensity. The wavelength $\lambda_{\text{max}}^{(\text{blue})}$ of blue light is shorter than the wavelength $\lambda_{\text{max}}^{(\text{red})}$ of red light. Even if we do not know the precise wavelengths, we can still set up a proportion.

Solution

Writing Wien's law for the blue star and for the red star, we have

$$\lambda_{\text{max}}^{(\text{red})} T_{(\text{red})} = 2.898 \times 10^{-3}\,\text{m} \cdot \text{K} = \lambda_{\text{max}}^{(\text{blue})} T_{(\text{blue})} \tag{6.2}$$

When simplified, **Equation 6.2** gives

$$T_{(\text{red})} = \frac{\lambda_{\text{max}}^{(\text{blue})}}{\lambda_{\text{max}}^{(\text{red})}} T_{(\text{blue})} < T_{(\text{blue})} \tag{6.3}$$

Therefore, Betelgeuse is cooler than Rigel.

Significance

Note that Wien's displacement law tells us that the higher the temperature of an emitting body, the shorter the wavelength of the radiation it emits. The qualitative analysis presented in this example is generally valid for any emitting body, whether it is a big object such as a star or a small object such as the glowing filament in an incandescent lightbulb.

6.1 Check Your Understanding The flame of a peach-scented candle has a yellowish color and the flame of a Bunsen's burner in a chemistry lab has a bluish color. Which flame has a higher temperature?

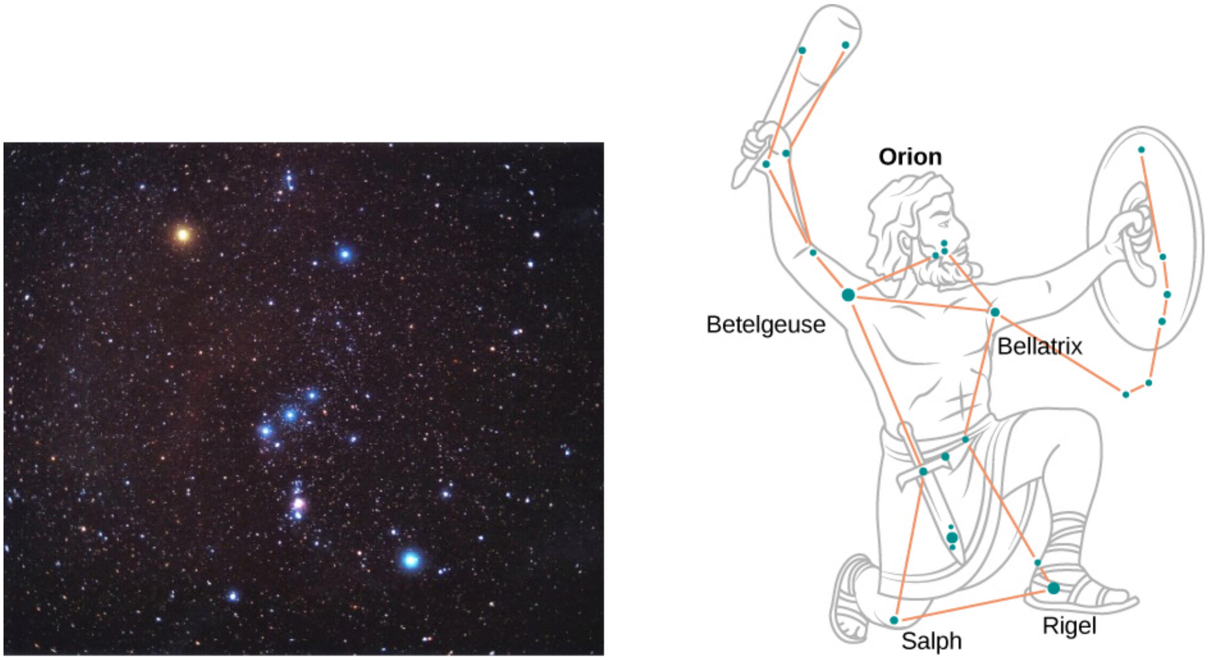


Figure 6.5 In the Orion constellation, the red star Betelgeuse, which usually takes on a yellowish tint, appears as the figure’s right shoulder (in the upper left). The giant blue star on the bottom right is Rigel, which appears as the hunter’s left foot. (credit left: modification of work by Matthew Spinelli, NASA APOD)

The second experimental relation is Stefan’s law**,** which concerns the total power of blackbody radiation emitted across the entire spectrum of wavelengths at a given temperature. In **Figure 6.3**, this total power is represented by the area under the blackbody radiation curve for a given T. As the temperature of a blackbody increases, the total emitted power also increases. Quantitatively, Stefan’s law expresses this relation as

$$P(T) = \sigma A T^4 \tag{6.4}$$

where A is the surface area of a blackbody, T is its temperature (in kelvins), and σ is the **Stefan–Boltzmann constant**, $\sigma = 5.670 \times 10^{-8}\,\text{W}/(\text{m}^2 \cdot \text{K}^4)$. Stefan’s law enables us to estimate how much energy a star is radiating by remotely measuring its temperature.

Example 6.2

Power Radiated by Stars

A star such as our Sun will eventually evolve to a “red giant” star and then to a “white dwarf” star. A typical white dwarf is approximately the size of Earth, and its surface temperature is about $2.5 \times 10^4\,\text{K}$. A typical red giant has a surface temperature of $3.0 \times 10^3\,\text{K}$ and a radius ~100,000 times larger than that of a white dwarf. What is the average radiated power per unit area and the total power radiated by each of these types of stars? How do they compare?

Strategy

If we treat the star as a blackbody, then according to Stefan’s law, the total power that the star radiates is proportional to the fourth power of its temperature. To find the power radiated per unit area of the surface, we do not need to make any assumptions about the shape of the star because P/A depends only on temperature. However, to compute the total power, we need to make an assumption that the energy radiates through a spherical surface enclosing the star, so that the surface area is $A = 4\pi R^2$, where R is its radius.

Solution

A simple proportion based on Stefan's law gives

$$\frac{P_{\text{dwarf}}/A_{\text{dwarf}}}{P_{\text{giant}}/A_{\text{giant}}} = \frac{\sigma T_{\text{dwarf}}^4}{\sigma T_{\text{giant}}^4} = \left(\frac{T_{\text{dwarf}}}{T_{\text{giant}}}\right)^4 = \left(\frac{2.5 \times 10^4}{3.0 \times 10^3}\right)^4 = 4820 \tag{6.5}$$

The power emitted per unit area by a white dwarf is about 5000 times that the power emitted by a red giant. Denoting this ratio by $a = 4.8 \times 10^3$, **Equation 6.5** gives

$$\frac{P_{\text{dwarf}}}{P_{\text{giant}}} = a\frac{A_{\text{dwarf}}}{A_{\text{giant}}} = a\frac{4\pi R_{\text{dwarf}}^2}{4\pi R_{\text{giant}}^2} = a\left(\frac{R_{\text{dwarf}}}{R_{\text{giant}}}\right)^2 = 4.8 \times 10^3 \left(\frac{R_{\text{dwarf}}}{10^5 R_{\text{dwarf}}}\right)^2 = 4.8 \times 10^{-7} \tag{6.6}$$

We see that the total power emitted by a white dwarf is a tiny fraction of the total power emitted by a red giant. Despite its relatively lower temperature, the overall power radiated by a red giant far exceeds that of the white dwarf because the red giant has a much larger surface area. To estimate the absolute value of the emitted power per unit area, we again use Stefan's law. For the white dwarf, we obtain

$$\frac{P_{\text{dwarf}}}{A_{\text{dwarf}}} = \sigma T_{\text{dwarf}}^4 = 5.670 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\left(2.5 \times 10^4\,\text{K}\right)^4 = 2.2 \times 10^{10}\,\text{W/m}^2 \tag{6.7}$$

The analogous result for the red giant is obtained by scaling the result for a white dwarf:

$$\frac{P_{\text{giant}}}{A_{\text{giant}}} = \frac{2.2 \times 10^{10}}{4.82 \times 10^3}\frac{\text{W}}{\text{m}^2} = 4.56 \times 10^6 \frac{\text{W}}{\text{m}^2} \cong 4.6 \times 10^6 \frac{\text{W}}{\text{m}^2} \tag{6.8}$$

Significance

To estimate the total power emitted by a white dwarf, in principle, we could use **Equation 6.7**. However, to find its surface area, we need to know the average radius, which is not given in this example. Therefore, the solution stops here. The same is also true for the red giant star.

6.2 Check Your Understanding An iron poker is being heated. As its temperature rises, the poker begins to glow—first dull red, then bright red, then orange, and then yellow. Use either the blackbody radiation curve or Wien's law to explain these changes in the color of the glow.

6.3 Check Your Understanding Suppose that two stars, α and β, radiate exactly the same total power. If the radius of star α is three times that of star β, what is the ratio of the surface temperatures of these stars? Which one is hotter?

The term "blackbody" was coined by Gustav R. Kirchhoff in 1862. The blackbody radiation curve was known experimentally, but its shape eluded physical explanation until the year 1900. The physical model of a blackbody at temperature T is that of the electromagnetic waves enclosed in a cavity (see **Figure 6.2**) and at thermodynamic equilibrium with the cavity walls. The waves can exchange energy with the walls. The objective here is to find the energy density distribution among various modes of vibration at various wavelengths (or frequencies). In other words, we want to know how much energy is carried by a single wavelength or a band of wavelengths. Once we know the energy distribution, we can use standard statistical methods (similar to those studied in a previous chapter) to obtain the blackbody radiation curve, Stefan's law, and Wien's displacement law. When the physical model is correct, the theoretical predictions should be the same as the experimental curves.

In a classical approach to the blackbody radiation problem, in which radiation is treated as waves (as you have studied in previous chapters), the modes of electromagnetic waves trapped in the cavity are in equilibrium and continually exchange their energies with the cavity walls. There is no physical reason why a wave should do otherwise: Any amount of energy can be exchanged, either by being transferred from the wave to the material in the wall or by being received by the wave from the material in the wall. This classical picture is the basis of the model developed by Lord Rayleigh and, independently, by Sir James Jeans. The result of this classical model for blackbody radiation curves is known as the *Rayleigh–Jeans law*.

However, as shown in **Figure 6.6**, the Rayleigh–Jeans law fails to correctly reproduce experimental results. In the limit of short wavelengths, the Rayleigh–Jeans law predicts infinite radiation intensity, which is inconsistent with the experimental results in which radiation intensity has finite values in the ultraviolet region of the spectrum. This divergence between the results of classical theory and experiments, which came to be called the *ultraviolet catastrophe*, shows how classical physics fails to explain the mechanism of blackbody radiation.

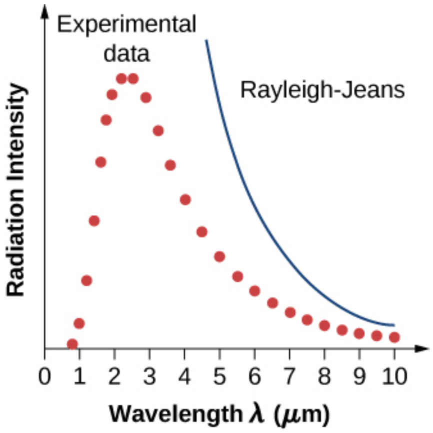


Figure 6.6 The ultraviolet catastrophe: The Rayleigh–Jeans law does not explain the observed blackbody emission spectrum.

The blackbody radiation problem was solved in 1900 by Max Planck. Planck used the same idea as the Rayleigh–Jeans model in the sense that he treated the electromagnetic waves between the walls inside the cavity classically, and assumed that the radiation is in equilibrium with the cavity walls. The innovative idea that Planck introduced in his model is the assumption that the cavity radiation originates from atomic oscillations inside the cavity walls, and that these oscillations can have only *discrete* values of energy. Therefore, the radiation trapped inside the cavity walls can exchange energy with the walls only in discrete amounts. Planck's hypothesis of discrete energy values, which he called *quanta*, assumes that the oscillators inside the cavity walls have **quantized energies**. This was a brand new idea that went beyond the classical physics of the nineteenth century because, as you learned in a previous chapter, in the classical picture, the energy of an oscillator can take on any continuous value. Planck assumed that the energy of an oscillator (E_n) can have only discrete, or quantized, values:

$$E_n = nhf, \quad \text{where } n = 1,\ 2,\ 3,\ ... \tag{6.9}$$

In **Equation 6.9**, *f* is the frequency of Planck's oscillator. The natural number *n* that enumerates these discrete energies is called a **quantum number**. The physical constant *h* is called *Planck's constant*:

$$h = 6.626 \times 10^{-34}\,\text{J}\cdot\text{s} = \ 4.136 \times 10^{-15}\,\text{eV}\cdot\text{s} \tag{6.10}$$

Each discrete energy value corresponds to a **quantum state of a Planck oscillator**. Quantum states are enumerated by quantum numbers. For example, when Planck's oscillator is in its first $n = 1$ quantum state, its energy is $E_1 = hf;$ when it is in the $n = 2$ quantum state, its energy is $E_2 = 2hf;$ when it is in the $n = 3$ quantum state, $E_3 = 3hf;$ and so on.

Note that **Equation 6.9** shows that there are infinitely many quantum states, which can be represented as a sequence {*hf*, 2*hf*, 3*hf*,…, (*n* – 1)*hf*, *nhf*, (*n* + 1)*hf*,…}. Each two consecutive quantum states in this sequence are separated by an energy jump, $\Delta E = hf.$ An oscillator in the wall can receive energy from the radiation in the cavity (absorption), or it can give away energy to the radiation in the cavity (emission). The absorption process sends the oscillator to a higher quantum state, and the emission process sends the oscillator to a lower quantum state. Whichever way this exchange of energy goes, the smallest amount of energy that can be exchanged is *hf*. There is no upper limit to how much energy can be exchanged, but whatever is exchanged must be an integer multiple of *hf*. If the energy packet does not have this exact amount, it is neither absorbed nor emitted at the wall of the blackbody.

Planck's Quantum Hypothesis

Planck's hypothesis of energy quanta states that the amount of energy emitted by the oscillator is carried by the quantum of radiation, ΔE :

$$\Delta E = hf$$

Recall that the frequency of electromagnetic radiation is related to its wavelength and to the speed of light by the fundamental relation $f\lambda = c$. This means that we can express **Equation 6.10** equivalently in terms of wavelength λ. When included in the computation of the energy density of a blackbody, Planck's hypothesis gives the following theoretical expression for the power intensity of emitted radiation per unit wavelength:

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda k_B T} - 1} \qquad (6.11)$$

where c is the speed of light in vacuum and k_B is Boltzmann's constant, $k_B = 1.380 \times 10^{-23}\,\text{J/K}$. The theoretical formula expressed in **Equation 6.11** is called *Planck's blackbody radiation law*. This law is in agreement with the experimental blackbody radiation curve (see **Figure 6.7**). In addition, Wien's displacement law and Stefan's law can both be derived from **Equation 6.11**. To derive Wien's displacement law, we use differential calculus to find the maximum of the radiation intensity curve $I(\lambda, T)$. To derive Stefan's law and find the value of the Stefan–Boltzmann constant, we use integral calculus and integrate $I(\lambda, T)$ to find the total power radiated by a blackbody at one temperature in the entire spectrum of wavelengths from $\lambda = 0$ to $\lambda = \infty$. This derivation is left as an exercise later in this chapter.

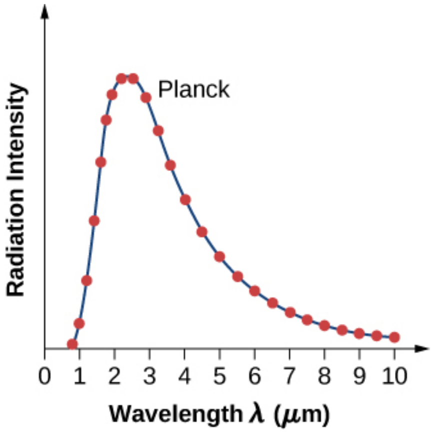


Figure 6.7 Planck's theoretical result (continuous curve) and the experimental blackbody radiation curve (dots).

Example 6.3

Planck's Quantum Oscillator

A quantum oscillator in the cavity wall in **Figure 6.2** is vibrating at a frequency of $5.0 \times 10^{14}\,\text{Hz}$. Calculate the spacing between its energy levels.

Strategy

Energy states of a quantum oscillator are given by **Equation 6.9**. The energy spacing ΔE is obtained by finding the energy difference between two adjacent quantum states for quantum numbers $n + 1$ and n.

Solution

We can substitute the given frequency and Planck's constant directly into the equation:

$$\Delta E = E_{n+1} - E_n = (n+1)hf - nhf = hf = (6.626 \times 10^{-34}\,\mathrm{J \cdot s})(5.0 \times 10^{14}\,\mathrm{Hz}) = 3.3 \times 10^{-19}\,\mathrm{J}$$

Significance

Note that we do not specify what kind of material was used to build the cavity. Here, a quantum oscillator is a theoretical model of an atom or molecule of material in the wall.

6.4 **Check Your Understanding** A molecule is vibrating at a frequency of $5.0 \times 10^{14}\,\mathrm{Hz}$. What is the smallest spacing between its vibrational energy levels?

Example 6.4

Quantum Theory Applied to a Classical Oscillator

A 1.0-kg mass oscillates at the end of a spring with a spring constant of 1000 N/m. The amplitude of these oscillations is 0.10 m. Use the concept of quantization to find the energy spacing for this classical oscillator. Is the energy quantization significant for macroscopic systems, such as this oscillator?

Strategy

We use **Equation 6.10** as though the system were a quantum oscillator, but with the frequency f of the mass vibrating on a spring. To evaluate whether or not quantization has a significant effect, we compare the quantum energy spacing with the macroscopic total energy of this classical oscillator.

Solution

For the spring constant, $k = 1.0 \times 10^3\,\mathrm{N/m}$, the frequency f of the mass, $m = 1.0\mathrm{kg}$, is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.0 \times 10^3\,\mathrm{N/m}}{1.0\mathrm{kg}}} \simeq 5.0\,\mathrm{Hz}$$

The energy quantum that corresponds to this frequency is

$$\Delta E = hf = (6.626 \times 10^{-34}\,\mathrm{J \cdot s})(5.0\mathrm{Hz}) = 3.3 \times 10^{-33}\,\mathrm{J}$$

When vibrations have amplitude $A = 0.10\mathrm{m}$, the energy of oscillations is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(1000\mathrm{N/m})(0.1\mathrm{m})^2 = 5.0\mathrm{J}$$

Significance

Thus, for a classical oscillator, we have $\Delta E/E \approx 10^{-34}$. We see that the separation of the energy levels is immeasurably small. Therefore, for all practical purposes, the energy of a classical oscillator takes on continuous values. This is why classical principles may be applied to macroscopic systems encountered in everyday life without loss of accuracy.

6.5 **Check Your Understanding** Would the result in **Example 6.4** be different if the mass were not 1.0 kg but a tiny mass of 1.0 μg, and the amplitude of vibrations were 0.10 μm?

When Planck first published his result, the hypothesis of energy quanta was not taken seriously by the physics community because it did not follow from any established physics theory at that time. It was perceived, even by Planck himself, as a useful mathematical trick that led to a good theoretical "fit" to the experimental curve. This perception was changed in 1905 when Einstein published his explanation of the photoelectric effect, in which he gave Planck's energy quantum a new meaning: that of a particle of light.

6.2 | Photoelectric Effect

Learning Objectives

By the end of this section you will be able to:

- Describe physical characteristics of the photoelectric effect
- Explain why the photoelectric effect cannot be explained by classical physics
- Describe how Einstein’s idea of a particle of radiation explains the photoelectric effect

When a metal surface is exposed to a monochromatic electromagnetic wave of sufficiently short wavelength (or equivalently, above a threshold frequency), the incident radiation is absorbed and the exposed surface emits electrons. This phenomenon is known as the **photoelectric effect**. Electrons that are emitted in this process are called **photoelectrons**.

The experimental setup to study the photoelectric effect is shown schematically in Figure 6.8. The target material serves as the anode, which becomes the emitter of photoelectrons when it is illuminated by monochromatic radiation. We call this electrode the **photoelectrode**. Photoelectrons are collected at the cathode, which is kept at a lower potential with respect to the anode. The potential difference between the electrodes can be increased or decreased, or its polarity can be reversed. The electrodes are enclosed in an evacuated glass tube so that photoelectrons do not lose their kinetic energy on collisions with air molecules in the space between electrodes.

When the target material is not exposed to radiation, no current is registered in this circuit because the circuit is broken (note, there is a gap between the electrodes). But when the target material is connected to the negative terminal of a battery and exposed to radiation, a current is registered in this circuit; this current is called the **photocurrent**. Suppose that we now reverse the potential difference between the electrodes so that the target material now connects with the positive terminal of a battery, and then we slowly increase the voltage. The photocurrent gradually dies out and eventually stops flowing completely at some value of this reversed voltage. The potential difference at which the photocurrent stops flowing is called the **stopping potential**.

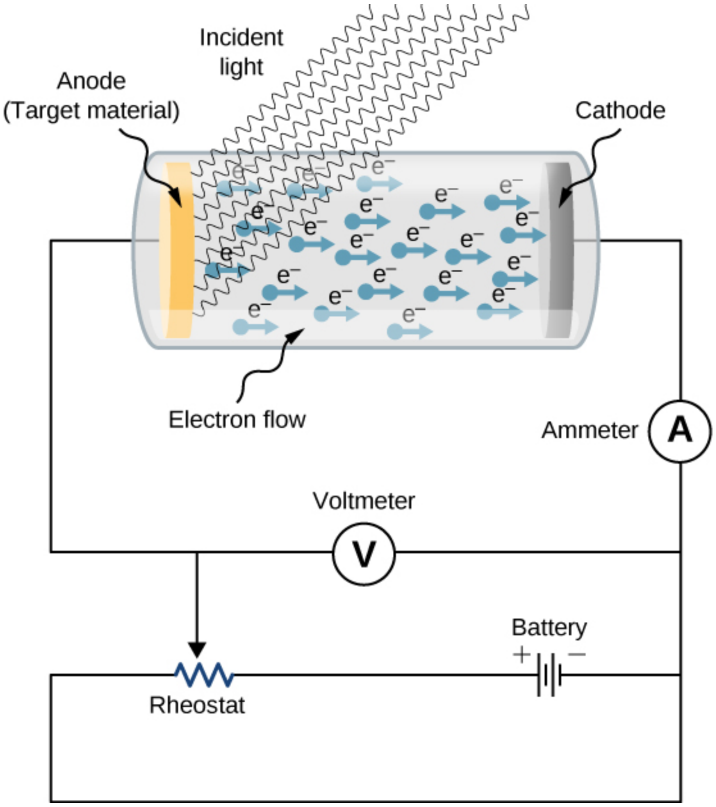


Figure 6.8 An experimental setup to study the photoelectric effect. The anode and cathode are enclosed in an evacuated glass tube. The voltmeter measures the electric potential difference between the electrodes, and the ammeter measures the photocurrent. The incident radiation is monochromatic.

Characteristics of the Photoelectric Effect

The photoelectric effect has three important characteristics that cannot be explained by classical physics: (1) the absence of a lag time, (2) the independence of the kinetic energy of photoelectrons on the intensity of incident radiation, and (3) the presence of a cut-off frequency. Let's examine each of these characteristics.

The absence of lag time

When radiation strikes the target material in the electrode, electrons are emitted almost instantaneously, even at very low intensities of incident radiation. This absence of lag time contradicts our understanding based on classical physics. Classical physics predicts that for low-energy radiation, it would take significant time before irradiated electrons could gain sufficient energy to leave the electrode surface; however, such an energy buildup is not observed.

The intensity of incident radiation and the kinetic energy of photoelectrons

Typical experimental curves are shown in **Figure 6.9**, in which the photocurrent is plotted versus the applied potential difference between the electrodes. For the positive potential difference, the current steadily grows until it reaches a plateau. Furthering the potential increase beyond this point does not increase the photocurrent at all. A higher intensity of radiation produces a higher value of photocurrent. For the negative potential difference, as the absolute value of the potential difference increases, the value of the photocurrent decreases and becomes zero at the stopping potential. For any intensity of incident radiation, whether the intensity is high or low, the value of the stopping potential always stays at one value.

To understand why this result is unusual from the point of view of classical physics, we first have to analyze the energy of photoelectrons. A photoelectron that leaves the surface has kinetic energy K. It gained this energy from the incident electromagnetic wave. In the space between the electrodes, a photoelectron moves in the electric potential and its energy changes by the amount $q\Delta V$, where ΔV is the potential difference and $q = -e$. Because no forces are present but electric force, by applying the work-energy theorem, we obtain the energy balance $\Delta K - e\Delta V = 0$ for the photoelectron, where ΔK is the change in the photoelectron's kinetic energy. When the stopping potential $-\Delta V_s$ is applied, the photoelectron loses its initial kinetic energy K_i and comes to rest. Thus, its energy balance becomes

$(0 - K_i) - e(-\Delta V_s) = 0,$ so that $K_i = e\Delta V_s.$ In the presence of the stopping potential, the largest kinetic energy K_{max} that a photoelectron can have is its initial kinetic energy, which it has at the surface of the photoelectrode. Therefore, the largest kinetic energy of photoelectrons can be directly measured by measuring the stopping potential:

$$K_{\text{max}} = e\Delta V_s. \tag{6.12}$$

At this point we can see where the classical theory is at odds with the experimental results. In classical theory, the photoelectron absorbs electromagnetic energy in a continuous way; this means that when the incident radiation has a high intensity, the kinetic energy in **Equation 6.12** is expected to be high. Similarly, when the radiation has a low intensity, the kinetic energy is expected to be low. But the experiment shows that the maximum kinetic energy of photoelectrons is independent of the light intensity.

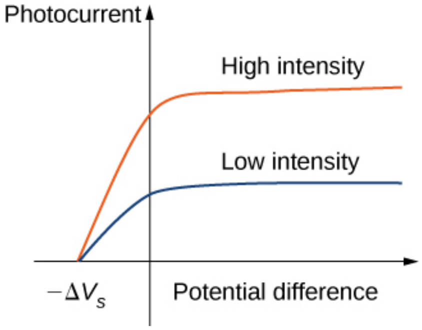


Figure 6.9 The detected photocurrent plotted versus the applied potential difference shows that for any intensity of incident radiation, whether the intensity is high (upper curve) or low (lower curve), the value of the stopping potential is always the same.

The presence of a cut-off frequency

For any metal surface, there is a minimum frequency of incident radiation below which photocurrent does not occur. The value of this **cut-off frequency** for the photoelectric effect is a physical property of the metal: Different materials have different values of cut-off frequency. Experimental data show a typical linear trend (see **Figure 6.10**). The kinetic energy of photoelectrons at the surface grows linearly with the increasing frequency of incident radiation. Measurements for all metal surfaces give linear plots with one slope. None of these observed phenomena is in accord with the classical understanding of nature. According to the classical description, the kinetic energy of photoelectrons should not depend on the frequency of incident radiation at all, and there should be no cut-off frequency. Instead, in the classical picture, electrons receive energy from the incident electromagnetic wave in a continuous way, and the amount of energy they receive depends only on the intensity of the incident light and nothing else. So in the classical understanding, as long as the light is shining, the photoelectric effect is expected to continue.

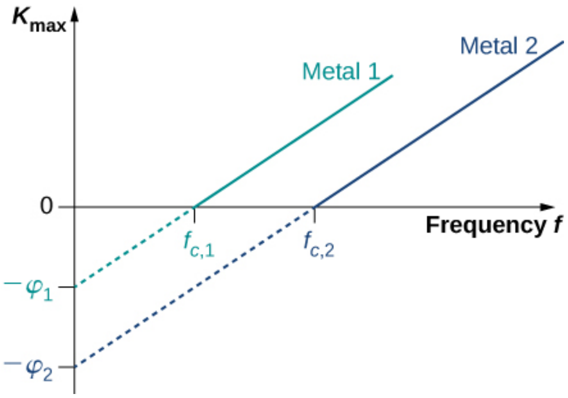


Figure 6.10 Kinetic energy of photoelectrons at the surface versus the frequency of incident radiation. The photoelectric effect can only occur above the cut-off frequency f_c.

Measurements for all metal surfaces give linear plots with one slope. Each metal surface has its own cut-off frequency.

The Work Function

The photoelectric effect was explained in 1905 by A. Einstein. Einstein reasoned that if Planck's hypothesis about energy quanta was correct for describing the energy exchange between electromagnetic radiation and cavity walls, it should also work to describe energy absorption from electromagnetic radiation by the surface of a photoelectrode. He postulated that an electromagnetic wave carries its energy in discrete packets. Einstein's postulate goes beyond Planck's hypothesis because it states that the light itself consists of energy quanta. In other words, it states that electromagnetic waves are quantized.

In Einstein's approach, a beam of monochromatic light of frequency *f* is made of photons. A **photon** is a particle of light. Each photon moves at the speed of light and carries an energy quantum E_f. A photon's energy depends only on its frequency *f*. Explicitly, the **energy of a photon** is

$$E_f = hf \tag{6.13}$$

where h is Planck's constant. In the photoelectric effect, photons arrive at the metal surface and each photon gives away *all* of its energy to only *one* electron on the metal surface. This transfer of energy from photon to electron is of the "all or nothing" type, and there are no fractional transfers in which a photon would lose only part of its energy and survive. The essence of a **quantum phenomenon** is either a photon transfers its entire energy and ceases to exist or there is no transfer at all. This is in contrast with the classical picture, where fractional energy transfers are permitted. Having this quantum understanding, the energy balance for an electron on the surface that receives the energy E_f from a photon is

$$E_f = K_{\max} + \phi$$

where $K_{\max}$ is the kinetic energy, given by **Equation 6.12**, that an electron has at the very instant it gets detached from the surface. In this energy balance equation, ϕ is the energy needed to detach a photoelectron from the surface. This energy ϕ is called the **work function** of the metal. Each metal has its characteristic work function, as illustrated in **Table 6.1**. To obtain the kinetic energy of photoelectrons at the surface, we simply invert the energy balance equation and use **Equation 6.13** to express the energy of the absorbed photon. This gives us the expression for the kinetic energy of photoelectrons, which explicitly depends on the frequency of incident radiation:

$$K_{\max} = hf - \phi. \tag{6.14}$$

This equation has a simple mathematical form but its physics is profound. We can now elaborate on the physical meaning behind **Equation 6.14**.

Typical Values of the Work Function for Some Common Metals

Metal	ϕ (eV)
Na	2.46
Al	4.08
Pb	4.14
Zn	4.31
Fe	4.50
Cu	4.70
Ag	4.73
Pt	6.35

Table 6.1

In Einstein’s interpretation, interactions take place between individual electrons and individual photons. The absence of a lag time means that these one-on-one interactions occur instantaneously. This interaction time cannot be increased by lowering the light intensity. The light intensity corresponds to the number of photons arriving at the metal surface per unit time. Even at very low light intensities, the photoelectric effect still occurs because the interaction is between one electron and one photon. As long as there is at least one photon with enough energy to transfer it to a bound electron, a photoelectron will appear on the surface of the photoelectrode.

The existence of the cut-off frequency f_c for the photoelectric effect follows from **Equation 6.14** because the kinetic energy K_{max} of the photoelectron can take only positive values. This means that there must be some threshold frequency for which the kinetic energy is zero, $0 = hf_c - \phi$. In this way, we obtain the explicit formula for cut-off frequency:

$$f_c = \frac{\phi}{h}. \tag{6.15}$$

Cut-off frequency depends only on the work function of the metal and is in direct proportion to it. When the work function is large (when electrons are bound fast to the metal surface), the energy of the threshold photon must be large to produce a photoelectron, and then the corresponding threshold frequency is large. Photons with frequencies larger than the threshold frequency f_c always produce photoelectrons because they have $K_{\text{max}} > 0$. Photons with frequencies smaller than f_c do not have enough energy to produce photoelectrons. Therefore, when incident radiation has a frequency below the cut-off frequency, the photoelectric effect is not observed. Because frequency f and wavelength λ of electromagnetic waves are related by the fundamental relation $\lambda f = c$ (where c is the speed of light in vacuum), the cut-off frequency has its corresponding **cut-off wavelength** λ_c :

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\phi / h} = \frac{hc}{\phi}. \tag{6.16}$$

In this equation, $hc = 1240\,\text{eV} \cdot \text{nm}$. Our observations can be restated in the following equivalent way: When the incident radiation has wavelengths longer than the cut-off wavelength, the photoelectric effect does not occur.

Example 6.5

Photoelectric Effect for Silver

Radiation with wavelength 300 nm is incident on a silver surface. Will photoelectrons be observed?

Strategy

Photoelectrons can be ejected from the metal surface only when the incident radiation has a shorter wavelength than the cut-off wavelength. The work function of silver is $\phi = 4.73\,\text{eV}$ (**Table 6.1**). To make the estimate, we use **Equation 6.16**.

Solution

The threshold wavelength for observing the photoelectric effect in silver is

$$\lambda_c = \frac{hc}{\phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{4.73\,\text{eV}} = 262\,\text{nm}.$$

The incident radiation has wavelength 300 nm, which is longer than the cut-off wavelength; therefore, photoelectrons are not observed.

Significance

If the photoelectrode were made of sodium instead of silver, the cut-off wavelength would be 504 nm and photoelectrons would be observed.

Equation 6.14 in Einstein's model tells us that the maximum kinetic energy of photoelectrons is a linear function of the frequency of incident radiation, which is illustrated in **Figure 6.10**. For any metal, the slope of this plot has a value of Planck's constant. The intercept with the K_{max} -axis gives us a value of the work function that is characteristic for the metal. On the other hand, K_{max} can be directly measured in the experiment by measuring the value of the stopping potential ΔV_s (see **Equation 6.12**) at which the photocurrent stops. These direct measurements allow us to determine experimentally the value of Planck's constant, as well as work functions of materials.

Einstein's model also gives a straightforward explanation for the photocurrent values shown in **Figure 6.9**. For example, doubling the intensity of radiation translates to doubling the number of photons that strike the surface per unit time. The larger the number of photons, the larger is the number of photoelectrons, which leads to a larger photocurrent in the circuit. This is how radiation intensity affects the photocurrent. The photocurrent must reach a plateau at some value of potential difference because, in unit time, the number of photoelectrons is equal to the number of incident photons and the number of incident photons does not depend on the applied potential difference at all, but only on the intensity of incident radiation. The stopping potential does not change with the radiation intensity because the kinetic energy of photoelectrons (see **Equation 6.14**) does not depend on the radiation intensity.

Example 6.6

Work Function and Cut-Off Frequency

When a 180-nm light is used in an experiment with an unknown metal, the measured photocurrent drops to zero at potential – 0.80 V. Determine the work function of the metal and its cut-off frequency for the photoelectric effect.

Strategy

To find the cut-off frequency f_c, we use **Equation 6.15**, but first we must find the work function ϕ. To find ϕ, we use **Equation 6.12** and **Equation 6.14**. Photocurrent drops to zero at the stopping value of potential, so we identify $\Delta V_s = 0.8\text{V}$.

Solution

We use **Equation 6.12** to find the kinetic energy of the photoelectrons:

$$K_{\text{max}} = e\Delta V_s = e(0.80\text{V}) = 0.80\,\text{eV}.$$

Now we solve **Equation 6.14** for ϕ :

$$\phi = hf - K_{\text{max}} = \frac{hc}{\lambda} - K_{\text{max}} = \frac{1240\,\text{eV}\cdot\text{nm}}{180\,\text{nm}} - 0.80\,\text{eV} = 6.09\,\text{eV}.$$

Finally, we use **Equation 6.15** to find the cut-off frequency:

$$f_c = \frac{\phi}{h} = \frac{6.09\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV} \cdot \text{s}} = 1.47 \times 10^{-15}\ \text{Hz}.$$

Significance

In calculations like the one shown in this example, it is convenient to use Planck's constant in the units of $\text{eV} \cdot \text{s}$ and express all energies in eV instead of joules.

Example 6.7

The Photon Energy and Kinetic Energy of Photoelectrons

A 430-nm violet light is incident on a calcium photoelectrode with a work function of 2.71 eV.

Find the energy of the incident photons and the maximum kinetic energy of ejected electrons.

Strategy

The energy of the incident photon is $E_f = hf = hc/\lambda,$ where we use $f\lambda = c.$ To obtain the maximum energy of the ejected electrons, we use **Equation 6.16**.

Solution

$$E_f = \frac{hc}{\lambda} = \frac{1240\ \text{eV} \cdot \text{nm}}{430\ \text{nm}} = 2.88\ \text{eV}, \ \ K_{\text{max}} = E_f - \phi = 2.88\ \text{eV} - 2.71\ \text{eV} = 0.17\ \text{eV}$$

Significance

In this experimental setup, photoelectrons stop flowing at the stopping potential of 0.17 V.

6.6 Check Your Understanding A yellow 589-nm light is incident on a surface whose work function is 1.20 eV. What is the stopping potential? What is the cut-off wavelength?

6.7 Check Your Understanding Cut-off frequency for the photoelectric effect in some materials is $8.0 \times 10^{13}\ \text{Hz}.$ When the incident light has a frequency of $1.2 \times 10^{14}\ \text{Hz},$ the stopping potential is measured as – 0.16 V. Estimate a value of Planck's constant from these data (in units $\text{J} \cdot \text{s}$ and $\text{eV} \cdot \text{s}$) and determine the percentage error of your estimation.

6.3 | The Compton Effect

Learning Objectives

By the end of this section, you will be able to:

- Describe Compton's experiment
- Explain the Compton wavelength shift
- Describe how experiments with X-rays confirm the particle nature of radiation

Two of Einstein's influential ideas introduced in 1905 were the theory of special relativity and the concept of a light quantum, which we now call a photon. Beyond 1905, Einstein went further to suggest that freely propagating electromagnetic waves consisted of photons that are particles of light in the same sense that electrons or other massive particles are particles of matter. A beam of monochromatic light of wavelength λ (or equivalently, of frequency *f*) can be seen either as a classical wave or as a collection of photons that travel in a vacuum with one speed, *c* (the speed of light), and all carrying the same energy, $E_f = hf$. This idea proved useful for explaining the interactions of light with particles of matter.

Momentum of a Photon

Unlike a particle of matter that is characterized by its rest mass m_0, a photon is massless. In a vacuum, unlike a particle of matter that may vary its speed but cannot reach the speed of light, a photon travels at only one speed, which is exactly the speed of light. From the point of view of Newtonian classical mechanics, these two characteristics imply that a photon should not exist at all. For example, how can we find the linear momentum or kinetic energy of a body whose mass is zero? This apparent paradox vanishes if we describe a photon as a relativistic particle. According to the theory of special relativity, any particle in nature obeys the relativistic energy equation

$$E^2 = p^2c^2 + m_0^2c^4. \tag{6.17}$$

This relation can also be applied to a photon. In **Equation 6.17**, E is the total energy of a particle, p is its linear momentum, and m_0 is its rest mass. For a photon, we simply set $m_0 = 0$ in this equation. This leads to the expression for the momentum p_f of a photon

$$p_f = \frac{E_f}{c}. \tag{6.18}$$

Here the photon's energy E_f is the same as that of a light quantum of frequency f, which we introduced to explain the photoelectric effect:

$$E_f = hf = \frac{hc}{\lambda}. \tag{6.19}$$

The wave relation that connects frequency f with wavelength λ and speed c also holds for photons:

$$\lambda f = c \tag{6.20}$$

Therefore, a photon can be equivalently characterized by either its energy and wavelength, or its frequency and momentum. **Equation 6.19** and **Equation 6.20** can be combined into the explicit relation between a photon's momentum and its wavelength:

$$p_f = \frac{h}{\lambda}. \tag{6.21}$$

Notice that this equation gives us only the magnitude of the photon's momentum and contains no information about the direction in which the photon is moving. To include the direction, it is customary to write the photon's momentum as a vector:

$$\vec{\mathbf{p}}_f = \hbar \vec{\mathbf{k}}. \tag{6.22}$$

In **Equation 6.22**, $\hbar = h/2\pi$ is the **reduced Planck's constant** (pronounced "h-bar"), which is just Planck's constant divided by the factor 2π. Vector $\vec{\mathbf{k}}$ is called the “wave vector” or propagation vector (the direction in which a photon is moving). The **propagation vector** shows the direction of the photon’s linear momentum vector. The magnitude of the wave vector is $k = \left| \vec{\mathbf{k}} \right| = 2\pi/\lambda$ and is called the **wave number**. Notice that this equation does not introduce any new physics. We can verify that the magnitude of the vector in **Equation 6.22** is the same as that given by **Equation 6.18**.

The Compton Effect

The **Compton effect** is the term used for an unusual result observed when X-rays are scattered on some materials. By classical theory, when an electromagnetic wave is scattered off atoms, the wavelength of the scattered radiation is expected to be the same as the wavelength of the incident radiation. Contrary to this prediction of classical physics, observations show that when X-rays are scattered off some materials, such as graphite, the scattered X-rays have different wavelengths from the wavelength of the incident X-rays. This classically unexplainable phenomenon was studied experimentally by Arthur H. Compton and his collaborators, and Compton gave its explanation in 1923.

To explain the shift in wavelengths measured in the experiment, Compton used Einstein’s idea of light as a particle. The Compton effect has a very important place in the history of physics because it shows that electromagnetic radiation cannot be explained as a purely wave phenomenon. The explanation of the Compton effect gave a convincing argument to the physics community that electromagnetic waves can indeed behave like a stream of photons, which placed the concept of a photon on firm ground.

The schematics of Compton’s experimental setup are shown in **Figure 6.11**. The idea of the experiment is straightforward: Monochromatic X-rays with wavelength λ are incident on a sample of graphite (the “target”), where they interact with atoms inside the sample; they later emerge as scattered X-rays with wavelength λ'. A detector placed behind the target can measure the intensity of radiation scattered in any direction θ with respect to the direction of the incident X-ray beam. This **scattering angle**, θ, is the angle between the direction of the scattered beam and the direction of the incident beam. In this experiment, we know the intensity and the wavelength λ of the incoming (incident) beam; and for a given scattering angle θ, we measure the intensity and the wavelength λ' of the outgoing (scattered) beam. Typical results of these measurements are shown in **Figure 6.12**, where the *x*-axis is the wavelength of the scattered X-rays and the *y*-axis is the intensity of the scattered X-rays, measured for different scattering angles (indicated on the graphs). For all scattering angles (except for $\theta = 0°$), we measure two intensity peaks. One peak is located at the wavelength λ, which is the wavelength of the incident beam. The other peak is located at some other wavelength, λ'. The two peaks are separated by $\Delta\lambda$, which depends on the scattering angle θ of the outgoing beam (in the direction of observation). The separation $\Delta\lambda$ is called the **Compton shift**.

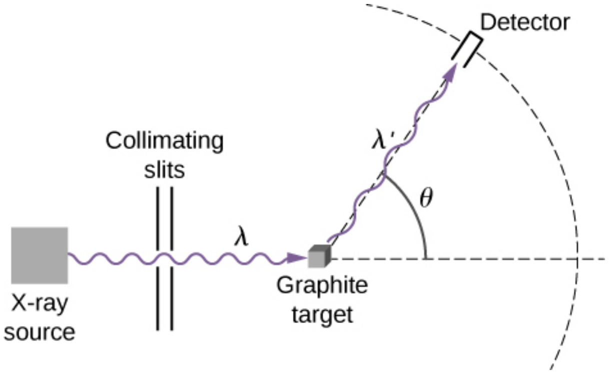


Figure 6.11 Experimental setup for studying Compton scattering.

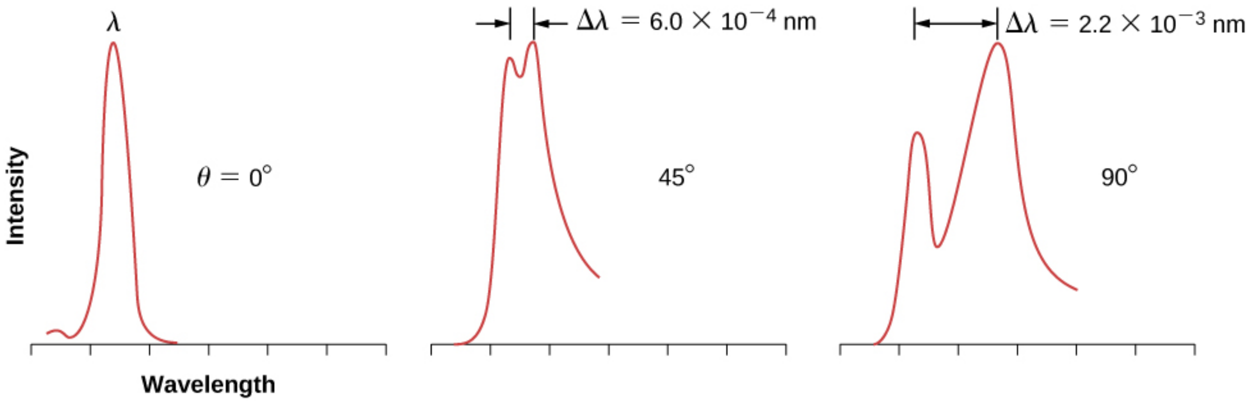


Figure 6.12 Experimental data show the Compton effect for X-rays scattering off graphite at various angles: The intensity of the scattered beam has two peaks. One peak appears at the wavelength λ of the incident radiation and the second peak appears at wavelength λ'. The separation $\Delta\lambda$ between the peaks depends on the scattering angle θ, which is the angular position of the detector in **Figure 6.11**. The experimental data in this figure are plotted in arbitrary units so that the height of the profile reflects the intensity of the scattered beam above background noise.

Compton Shift

As given by Compton, the explanation of the Compton shift is that in the target material, graphite, valence electrons are loosely bound in the atoms and behave like free electrons. Compton assumed that the incident X-ray radiation is a stream of photons. An incoming photon in this stream collides with a valence electron in the graphite target. In the course of this collision, the incoming photon transfers some part of its energy and momentum to the target electron and leaves the scene as a scattered photon. This model explains in qualitative terms why the scattered radiation has a longer wavelength than the incident radiation. Put simply, a photon that has lost some of its energy emerges as a photon with a lower frequency, or equivalently, with a longer wavelength. To show that his model was correct, Compton used it to derive the expression for the Compton shift. In his derivation, he assumed that both photon and electron are relativistic particles and that the collision obeys two commonsense principles: (1) the conservation of linear momentum and (2) the conservation of total relativistic energy.

In the following derivation of the Compton shift, E_f and $\vec{\mathbf{p}}_f$ denote the energy and momentum, respectively, of an incident photon with frequency *f*. The photon collides with a relativistic electron at rest, which means that immediately before the collision, the electron's energy is entirely its rest mass energy, $m_0 c^2$. Immediately after the collision, the electron has energy *E* and momentum $\vec{\mathbf{p}}$, both of which satisfy **Equation 6.19**. Immediately after the collision, the outgoing photon has energy $\tilde{E}_f$, momentum $\vec{\tilde{\mathbf{p}}}_f$, and frequency f'. The direction of the incident photon is horizontal from left to right, and the direction of the outgoing photon is at the angle θ, as illustrated in **Figure 6.11**. The scattering angle θ is the angle between the momentum vectors $\vec{\mathbf{p}}_f$ and $\vec{\tilde{\mathbf{p}}}_f$, and we can write their scalar product:

$$\vec{\mathbf{p}}_f \cdot \vec{\tilde{\mathbf{p}}}_f = p_f \tilde{p}_f \cos\theta. \tag{6.23}$$

Following Compton's argument, we assume that the colliding photon and electron form an isolated system. This assumption is valid for weakly bound electrons that, to a good approximation, can be treated as free particles. Our first equation is the conservation of energy for the photon-electron system:

$$E_f + m_0 c^2 = \tilde{E}_f + E. \tag{6.24}$$

The left side of this equation is the energy of the system at the instant immediately before the collision, and the right side of the equation is the energy of the system at the instant immediately after the collision. Our second equation is the conservation of linear momentum for the photon–electron system where the electron is at rest at the instant immediately before the collision:

$$\vec{\mathbf{p}}_f = \vec{\tilde{\mathbf{p}}}_f + \vec{\mathbf{p}}\,. \tag{6.25}$$

The left side of this equation is the momentum of the system right before the collision, and the right side of the equation is the momentum of the system right after collision. The entire physics of Compton scattering is contained in these three preceding equations—the remaining part is algebra. At this point, we could jump to the concluding formula for the Compton shift, but it is beneficial to highlight the main algebraic steps that lead to Compton's formula, which we give here as follows.

We start with rearranging the terms in **Equation 6.24** and squaring it:

$$\left[\left(E_f - \tilde{E}_f\right) + m_0 c^2\right]^2 = E^2.$$

In the next step, we substitute **Equation 6.19** for $E^2,$ simplify, and divide both sides by c^2 to obtain

$$\left(E_f / c - \tilde{E}_f / c\right)^2 + 2m_0 c\left(E_f / c - \tilde{E}_f / c\right) = p^2.$$

Now we can use **Equation 6.21** to express this form of the energy equation in terms of momenta. The result is

$$\left(p_f - \tilde{p}_f\right)^2 + 2m_0 c\left(p_f - \tilde{p}_f\right) = p^2. \tag{6.26}$$

To eliminate $p^2,$ we turn to the momentum equation **Equation 6.25**, rearrange its terms, and square it to obtain

$$\left(\vec{\mathbf{p}}_f - \vec{\tilde{\mathbf{p}}}_f\right)^2 = p^2 \text{ and } \left(\vec{\mathbf{p}}_f - \vec{\tilde{\mathbf{p}}}_f\right)^2 = p_f^2 + \tilde{p}_f^2 - 2\,\vec{\mathbf{p}}_f \cdot \vec{\tilde{\mathbf{p}}}_f.$$

The product of the momentum vectors is given by **Equation 6.23**. When we substitute this result for p^2 in **Equation 6.26**, we obtain the energy equation that contains the scattering angle θ :

$$\left(p_f - \tilde{p}_f\right)^2 + 2m_0 c\left(p_f - \tilde{p}_f\right) = p_f^2 + \tilde{p}_f^2 - 2p_f \tilde{p}_f \cos\theta.$$

With further algebra, this result can be simplified to

$$\frac{1}{\tilde{p}_f} - \frac{1}{p_f} = \frac{1}{m_0 c}(1 - \cos\theta). \tag{6.27}$$

Now recall **Equation 6.21** and write: $1/\tilde{p}_f = \lambda' / h$ and $1/p_f = \lambda / h.$ When these relations are substituted into **Equation 6.27**, we obtain the relation for the Compton shift:

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta). \tag{6.28}$$

The factor $h/m_0 c$ is called the **Compton wavelength** of the electron:

$$\lambda_c = \frac{h}{m_0 c} = 0.00243\text{ nm} = 2.43\text{ pm}. \tag{6.29}$$

Denoting the shift as $\Delta\lambda = \lambda' - \lambda,$ the concluding result can be rewritten as

$$\Delta\lambda = \lambda_c(1 - \cos\theta). \tag{6.30}$$

This formula for the Compton shift describes outstandingly well the experimental results shown in **Figure 6.12**. Scattering data measured for molybdenum, graphite, calcite, and many other target materials are in accord with this theoretical result. The nonshifted peak shown in **Figure 6.12** is due to photon collisions with tightly bound inner electrons in the target material. Photons that collide with the inner electrons of the target atoms in fact collide with the entire atom. In this extreme case, the rest mass in **Equation 6.29** must be changed to the rest mass of the atom. This type of shift is four orders of magnitude smaller than the shift caused by collisions with electrons and is so small that it can be neglected.

Compton scattering is an example of **inelastic scattering**, in which the scattered radiation has a longer wavelength than the wavelength of the incident radiation. In today's usage, the term "Compton scattering" is used for the inelastic scattering of photons by free, charged particles. In Compton scattering, treating photons as particles with momenta that can be transferred to charged particles provides the theoretical background to explain the wavelength shifts measured in experiments; this is the evidence that radiation consists of photons.

Example 6.8

Compton Scattering

An incident 71-pm X-ray is incident on a calcite target. Find the wavelength of the X-ray scattered at a $30°$ angle. What is the largest shift that can be expected in this experiment?

Strategy

To find the wavelength of the scattered X-ray, first we must find the Compton shift for the given scattering angle, $\theta = 30°$. We use **Equation 6.30**. Then we add this shift to the incident wavelength to obtain the scattered wavelength. The largest Compton shift occurs at the angle θ when $1 - \cos\theta$ has the largest value, which is for the angle $\theta = 180°$.

Solution

The shift at $\theta = 30°$ is

$$\Delta\lambda = \lambda_c(1 - \cos 30°) = 0.134\lambda_c = (0.134)(2.43)\text{ pm} = 0.325\text{ pm}.$$

This gives the scattered wavelength:

$$\lambda' = \lambda + \Delta\lambda = (71 + 0.325)\text{ pm} = 71.325\text{ pm}.$$

The largest shift is

$$(\Delta\lambda)_{\text{max}} = \lambda_c(1 - \cos 180^{0}) = 2(2.43\text{ pm}) = 4.86\text{ pm}.$$

Significance

The largest shift in wavelength is detected for the backscattered radiation; however, most of the photons from the incident beam pass through the target and only a small fraction of photons gets backscattered (typically, less than 5%). Therefore, these measurements require highly sensitive detectors.

6.8 Check Your Understanding An incident 71-pm X-ray is incident on a calcite target. Find the wavelength of the X-ray scattered at a $60°$ angle. What is the smallest shift that can be expected in this experiment?

6.4 | Bohr's Model of the Hydrogen Atom

Learning Objectives

By the end of this section, you will be able to:

- Explain the difference between the absorption spectrum and the emission spectrum of radiation emitted by atoms
- Describe the Rutherford gold foil experiment and the discovery of the atomic nucleus
- Explain the atomic structure of hydrogen
- Describe the postulates of the early quantum theory for the hydrogen atom
- Summarize how Bohr's quantum model of the hydrogen atom explains the radiation spectrum of atomic hydrogen

Historically, Bohr's model of the hydrogen atom is the very first model of atomic structure that correctly explained the radiation spectra of atomic hydrogen. The model has a special place in the history of physics because it introduced an early quantum theory, which brought about new developments in scientific thought and later culminated in the development of quantum mechanics. To understand the specifics of Bohr's model, we must first review the nineteenth-century discoveries that prompted its formulation.

When we use a prism to analyze white light coming from the sun, several dark lines in the solar spectrum are observed (**Figure 6.13**). Solar absorption lines are called **Fraunhofer lines** after Joseph von Fraunhofer, who accurately measured their wavelengths. During 1854–1861, Gustav Kirchhoff and Robert Bunsen discovered that for the various chemical elements, the line **emission spectrum** of an element exactly matches its line **absorption spectrum**. The difference between the absorption spectrum and the emission spectrum is explained in **Figure 6.14**. An absorption spectrum is observed when light passes through a gas. This spectrum appears as black lines that occur only at certain wavelengths on the background of the continuous spectrum of white light (**Figure 6.13**). The missing wavelengths tell us which wavelengths of the radiation are absorbed by the gas. The emission spectrum is observed when light is emitted by a gas. This spectrum is seen as colorful lines on the black background (see **Figure 6.15** and **Figure 6.16**). Positions of the emission lines tell us which wavelengths of the radiation are emitted by the gas. Each chemical element has its own characteristic emission spectrum. For each element, the positions of its emission lines are exactly the same as the positions of its absorption lines. This means that atoms of a specific element absorb radiation only at specific wavelengths and radiation that does not have these wavelengths is not absorbed by the element at all. This also means that the radiation emitted by atoms of each element has exactly the same wavelengths as the radiation they absorb.

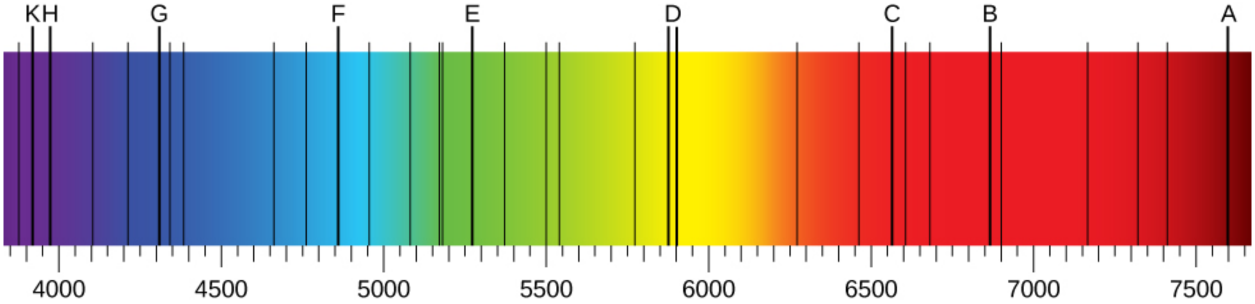


Figure 6.13 In the solar emission spectrum in the visible range from 380 nm to 710 nm, Fraunhofer lines are observed as vertical black lines at specific spectral positions in the continuous spectrum. Highly sensitive modern instruments observe thousands of such lines.

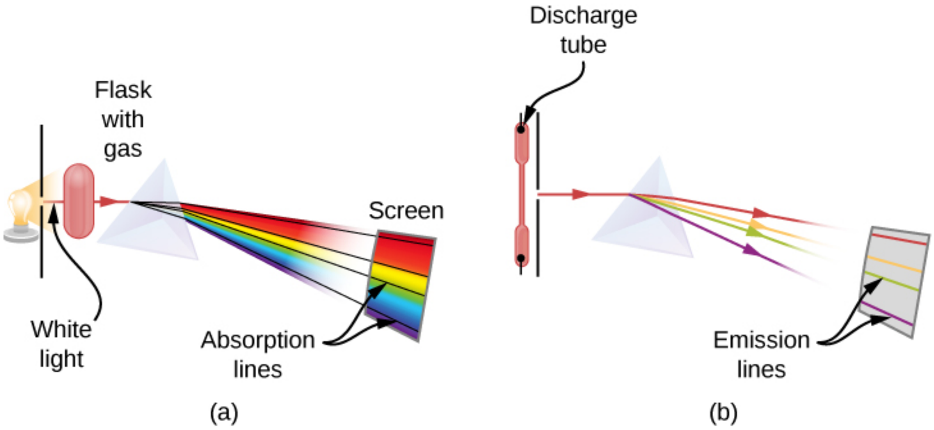


Figure 6.14 Observation of line spectra: (a) setup to observe absorption lines; (b) setup to observe emission lines. (a) White light passes through a cold gas that is contained in a glass flask. A prism is used to separate wavelengths of the passed light. In the spectrum of the passed light, some wavelengths are missing, which are seen as black absorption lines in the continuous spectrum on the viewing screen. (b) A gas is contained in a glass discharge tube that has electrodes at its ends. At a high potential difference between the electrodes, the gas glows and the light emitted from the gas passes through the prism that separates its wavelengths. In the spectrum of the emitted light, only specific wavelengths are present, which are seen as colorful emission lines on the screen.

Figure 6.15 The emission spectrum of atomic hydrogen: The spectral positions of emission lines are characteristic for hydrogen atoms. (credit: "Merikanto"/Wikimedia Commons)

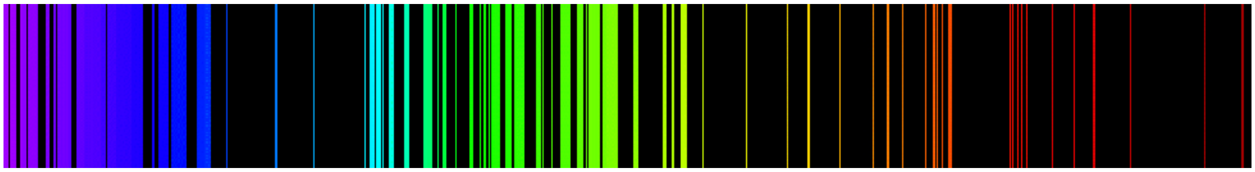


Figure 6.16 The emission spectrum of atomic iron: The spectral positions of emission lines are characteristic for iron atoms.

Emission spectra of the elements have complex structures; they become even more complex for elements with higher atomic numbers. The simplest spectrum, shown in **Figure 6.15**, belongs to the hydrogen atom. Only four lines are visible to the human eye. As you read from right to left in **Figure 6.15**, these lines are: red (656 nm), called the H-α line; aqua (486 nm), blue (434 nm), and violet (410 nm). The lines with wavelengths shorter than 400 nm appear in the ultraviolet part of the spectrum (**Figure 6.15**, far left) and are invisible to the human eye. There are infinitely many invisible spectral lines in the series for hydrogen.

An empirical formula to describe the positions (wavelengths) λ of the hydrogen emission lines in this series was discovered in 1885 by Johann Balmer. It is known as the **Balmer formula**:

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{2^2} - \frac{1}{n^2}\right). \tag{6.31}$$

The constant $R_{\mathrm{H}} = 1.09737 \times 10^7\ \mathrm{m}^{-1}$ is called the **Rydberg constant for hydrogen**. In **Equation 6.31**, the positive integer n takes on values $n = 3, 4, 5, 6$ for the four visible lines in this series. The series of emission lines given by the Balmer formula is called the **Balmer series** for hydrogen. Other emission lines of hydrogen that were discovered in the twentieth century are described by the **Rydberg formula**, which summarizes all of the experimental data:

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), \text{ where } n_i = n_f + 1, n_f + 2, n_f + 3, \ldots \tag{6.32}$$

When $n_f = 1,$ the series of spectral lines is called the **Lyman series**. When $n_f = 2,$ the series is called the Balmer series, and in this case, the Rydberg formula coincides with the Balmer formula. When $n_f = 3,$ the series is called the **Paschen series**. When $n_f = 4,$ the series is called the **Brackett series**. When $n_f = 5,$ the series is called the **Pfund series**. When $n_f = 6,$ we have the **Humphreys series**. As you may guess, there are infinitely many such spectral bands in the spectrum of hydrogen because n_f can be any positive integer number.

The Rydberg formula for hydrogen gives the exact positions of the spectral lines as they are observed in a laboratory; however, at the beginning of the twentieth century, nobody could explain why it worked so well. The Rydberg formula remained unexplained until the first successful model of the hydrogen atom was proposed in 1913.

Example 6.9

Limits of the Balmer Series

Calculate the longest and the shortest wavelengths in the Balmer series.

Strategy

We can use either the Balmer formula or the Rydberg formula. The longest wavelength is obtained when $1/n_i$ is largest, which is when $n_i = n_f + 1 = 3,$ because $n_f = 2$ for the Balmer series. The smallest wavelength is obtained when $1/n_i$ is smallest, which is $1/n_i \to 0$ when $n_i \to \infty$.

Solution

The long-wave limit:

$$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = (1.09737 \times 10^7)\frac{1}{\text{m}}\left(\frac{1}{4} - \frac{1}{9}\right) \Rightarrow \lambda = 656.3\,\text{nm}$$

The short-wave limit:

$$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{2^2} - 0\right) = (1.09737 \times 10^7)\frac{1}{\text{m}}\left(\frac{1}{4}\right) \Rightarrow \lambda = 364.6\,\text{nm}$$

Significance

Note that there are infinitely many spectral lines lying between these two limits.

6.9 **Check Your Understanding** What are the limits of the Lyman series? Can you see these spectral lines?

The key to unlocking the mystery of atomic spectra is in understanding atomic structure. Scientists have long known that matter is made of atoms. According to nineteenth-century science, atoms are the smallest indivisible quantities of matter. This scientific belief was shattered by a series of groundbreaking experiments that proved the existence of subatomic particles, such as electrons, protons, and neutrons.

The electron was discovered and identified as the smallest quantity of electric charge by J.J. Thomson in 1897 in his cathode ray experiments, also known as β-ray experiments: A **β-ray** is a beam of electrons. In 1904, Thomson proposed the first model of atomic structure, known as the “plum pudding” model, in which an atom consisted of an unknown positively charged matter with negative electrons embedded in it like plums in a pudding. Around 1900, E. Rutherford, and independently, Paul Ulrich Villard, classified all radiation known at that time as **α-rays**, β-rays, and **γ-rays** (a γ-ray is a beam of highly energetic photons). In 1907, Rutherford and Thomas Royds used spectroscopy methods to show that positively charged particles of α-radiation (called **α-particles**) are in fact doubly ionized atoms of helium. In 1909, Rutherford, Ernest Marsden, and Hans Geiger used α-particles in their famous scattering experiment that disproved Thomson’s model (see **Linear Momentum and Collisions (http://cnx.org/content/m58317/latest/)**).

In the **Rutherford gold foil experiment** (also known as the Geiger–Marsden experiment), α-particles were incident on a thin gold foil and were scattered by gold atoms inside the foil (see **Types of Collisions (http://cnx.org/content/m58321/latest/#CNX_UPhysics_09_04_TvsR)**). The outgoing particles were detected by a $360°$ scintillation screen surrounding the gold target (for a detailed description of the experimental setup, see **Linear Momentum and Collisions (http://cnx.org/content/m58317/latest/)**). When a scattered particle struck the screen, a tiny flash of light (scintillation) was observed at that location. By counting the scintillations seen at various angles with respect to the direction of the incident beam, the scientists could determine what fraction of the incident particles were scattered and what fraction were not deflected at all. If the plum pudding model were correct, there would be no back-scattered α-particles. However, the results of the Rutherford experiment showed that, although a sizable fraction of α-particles emerged from the foil not scattered at all as though the foil were not in their way, a significant fraction of α-particles were back-scattered toward the source. This kind of result was possible only when most of the mass and the entire positive charge of the gold atom were concentrated in a tiny space inside the atom.

In 1911, Rutherford proposed a **nuclear model of the atom**. In Rutherford’s model, an atom contained a positively charged nucleus of negligible size, almost like a point, but included almost the entire mass of the atom. The atom also contained negative electrons that were located within the atom but relatively far away from the nucleus. Ten years later, Rutherford coined the name *proton* for the nucleus of hydrogen and the name *neutron* for a hypothetical electrically neutral particle that would mediate the binding of positive protons in the nucleus (the neutron was discovered in 1932 by James Chadwick). Rutherford is credited with the discovery of the atomic nucleus; however, the Rutherford model of atomic structure does not explain the Rydberg formula for the hydrogen emission lines.

Bohr's model of the hydrogen atom, proposed by Niels Bohr in 1913, was the first quantum model that correctly explained the hydrogen emission spectrum. Bohr's model combines the classical mechanics of planetary motion with the quantum concept of photons. Once Rutherford had established the existence of the atomic nucleus, Bohr's intuition that the negative electron in the hydrogen atom must revolve around the positive nucleus became a logical consequence of the inverse-square-distance law of electrostatic attraction. Recall that Coulomb's law describing the attraction between two opposite charges has a similar form to Newton's universal law of gravitation in the sense that the gravitational force and the electrostatic force are both decreasing as $1/r^2$, where r is the separation distance between the bodies. In the same way as Earth revolves around the sun, the negative electron in the hydrogen atom can revolve around the positive nucleus. However, an accelerating charge radiates its energy. Classically, if the electron moved around the nucleus in a planetary fashion, it would be undergoing centripetal acceleration, and thus would be radiating energy that would cause it to spiral down into the nucleus. Such a planetary hydrogen atom would not be stable, which is contrary to what we know about ordinary hydrogen atoms that do not disintegrate. Moreover, the classical motion of the electron is not able to explain the discrete emission spectrum of hydrogen.

To circumvent these two difficulties, Bohr proposed the following three **postulates of Bohr's model**:

1. The negative electron moves around the positive nucleus (proton) in a circular orbit. All electron orbits are centered at the nucleus. Not all classically possible orbits are available to an electron bound to the nucleus.
2. The allowed electron orbits satisfy the *first quantization condition*: In the *n*th orbit, the angular momentum L_n of the electron can take only discrete values:

$$L_n = n\hbar, \text{ where } n = 1, 2, 3, \ldots \tag{6.33}$$

   This postulate says that the electron's angular momentum is quantized. Denoted by r_n and v_n, respectively, the radius of the *n*th orbit and the electron's speed in it, the first quantization condition can be expressed explicitly as

$$m_e v_n r_n = n\hbar. \tag{6.34}$$

3. An electron is allowed to make transitions from one orbit where its energy is E_n to another orbit where its energy is E_m. When an atom absorbs a photon, the electron makes a transition to a higher-energy orbit. When an atom emits a photon, the electron transits to a lower-energy orbit. Electron transitions with the simultaneous photon absorption or photon emission take place *instantaneously*. The allowed electron transitions satisfy the *second quantization condition*:

$$hf = |E_n - E_m| \tag{6.35}$$

   where hf is the energy of either an emitted or an absorbed photon with frequency *f*. The second quantization condition states that an electron's change in energy in the hydrogen atom is quantized.

These three postulates of the early quantum theory of the hydrogen atom allow us to derive not only the Rydberg formula, but also the value of the Rydberg constant and other important properties of the hydrogen atom such as its energy levels, its ionization energy, and the sizes of electron orbits. Note that in Bohr's model, along with two nonclassical quantization postulates, we also have the classical description of the electron as a particle that is subjected to the Coulomb force, and its motion must obey Newton's laws of motion. The hydrogen atom, as an isolated system, must obey the laws of conservation of energy and momentum in the way we know from classical physics. Having this theoretical framework in mind, we are ready to proceed with our analysis.

Electron Orbits

To obtain the size r_n of the electron's *n*th orbit and the electron's speed v_n in it, we turn to Newtonian mechanics. As a charged particle, the electron experiences an electrostatic pull toward the positively charged nucleus in the center of its circular orbit. This electrostatic pull is the centripetal force that causes the electron to move in a circle around the nucleus. Therefore, the magnitude of centripetal force is identified with the magnitude of the electrostatic force:

$$\frac{m_e v_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2}. \tag{6.36}$$

Here, e denotes the value of the elementary charge. The negative electron and positive proton have the same value of charge, $|q| = e.$ When **Equation 6.36** is combined with the first quantization condition given by **Equation 6.34**, we can solve for the speed, $v_n,$ and for the radius, r_n :

$$v_n = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar}\frac{1}{n} \tag{6.37}$$

$$r_n = 4\pi\varepsilon_0\frac{\hbar^2}{m_e e^2}n^2. \tag{6.38}$$

Note that these results tell us that the electron's speed as well as the radius of its orbit depend only on the index *n* that enumerates the orbit because all other quantities in the preceding equations are fundamental constants. We see from **Equation 6.38** that the size of the orbit grows as the square of *n*. This means that the second orbit is four times as large as the first orbit, and the third orbit is nine times as large as the first orbit, and so on. We also see from **Equation 6.37** that the electron's speed in the orbit decreases as the orbit size increases. The electron's speed is largest in the first Bohr orbit, for $n = 1,$ which is the orbit closest to the nucleus. The radius of the first Bohr orbit is called the **Bohr radius of hydrogen**, denoted as $a_0.$ Its value is obtained by setting $n = 1$ in **Equation 6.38**:

$$a_0 = 4\pi\varepsilon_0\frac{\hbar^2}{m_e e^2} = 5.29 \times 10^{-11}\,\text{m} = 0.529\,\text{Å}. \tag{6.39}$$

We can substitute a_0 in **Equation 6.38** to express the radius of the *n*th orbit in terms of a_0 :

$$r_n = a_0 n^2. \tag{6.40}$$

This result means that the electron orbits in hydrogen atom are *quantized* because the orbital radius takes on only specific values of $a_0, 4a_0, 9a_0, 16a_0, \ldots$ given by **Equation 6.40**, and no other values are allowed.

Electron Energies

The total energy E_n of an electron in the *n*th orbit is the sum of its kinetic energy K_n and its electrostatic potential energy $U_n.$ Utilizing **Equation 6.37**, we find that

$$K_n = \frac{1}{2}m_e v_n^2 = \frac{1}{32\pi^2\varepsilon_0^2}\frac{m_e e^4}{\hbar^2}\frac{1}{n^2}. \tag{6.41}$$

Recall that the electrostatic potential energy of interaction between two charges q_1 and q_2 that are separated by a distance r_{12} is $(1/4\pi\varepsilon_0)q_1 q_2/r_{12}.$ Here, $q_1 = +e$ is the charge of the nucleus in the hydrogen atom (the charge of the proton), $q_2 = -e$ is the charge of the electron and $r_{12} = r_n$ is the radius of the *n*th orbit. Now we use **Equation 6.38** to find the potential energy of the electron:

$$U_n = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{1}{16\pi^2\varepsilon_0^2}\frac{m_e e^4}{\hbar^2}\frac{1}{n^2}. \quad (6.42)$$

The total energy of the electron is the sum of Equation 6.41 and Equation 6.42:

$$E_n = K_n + U_n = -\frac{1}{32\pi^2\varepsilon_0^2}\frac{m_e e^4}{\hbar^2}\frac{1}{n^2}. \quad (6.43)$$

Note that the energy depends only on the index *n* because the remaining symbols in Equation 6.43 are physical constants. The value of the constant factor in Equation 6.43 is

$$E_0 = \frac{1}{32\pi^2\varepsilon_0^2}\frac{m_e e^4}{\hbar^2} = \frac{1}{8\varepsilon_0^2}\frac{m_e e^4}{h^2} = 2.17\times 10^{-18}\,\text{J} = 13.6\,\text{eV}. \quad (6.44)$$

It is convenient to express the electron's energy in the *n*th orbit in terms of this energy, as

$$E_n = -E_0\frac{1}{n^2}. \quad (6.45)$$

Now we can see that the electron energies in the hydrogen atom are *quantized* because they can have only discrete values of $-E_0,\ -E_0/4,\ -E_0/9,\ -E_0/16,\ \ldots$ given by Equation 6.45, and no other energy values are allowed. This set of allowed electron energies is called the **energy spectrum of hydrogen** (Figure 6.17). The index *n* that enumerates energy levels in Bohr's model is called the energy **quantum number**. We identify the energy of the electron inside the hydrogen atom with the energy of the hydrogen atom. Note that the smallest value of energy is obtained for $n = 1$, so the hydrogen atom cannot have energy smaller than that. This smallest value of the electron energy in the hydrogen atom is called the **ground state energy of the hydrogen atom** and its value is

$$E_1 = -E_0 = -13.6\,\text{eV}. \quad (6.46)$$

The hydrogen atom may have other energies that are higher than the ground state. These higher energy states are known as **excited energy states of a hydrogen atom**.

There is only one ground state, but there are infinitely many excited states because there are infinitely many values of *n* in Equation 6.45. We say that the electron is in the "first exited state" when its energy is E_2 (when $n = 2$), the second excited state when its energy is E_3 (when $n = 3$) and, in general, in the *n*th exited state when its energy is E_{n+1}. There is no highest-of-all excited state; however, there is a limit to the sequence of excited states. If we keep increasing *n* in Equation 6.45, we find that the limit is $-\lim_{n\to\infty} E_0/n^2 = 0$. In this limit, the electron is no longer bound to the nucleus but becomes a free electron. An electron remains bound in the hydrogen atom as long as its energy is negative. An electron that orbits the nucleus in the first Bohr orbit, closest to the nucleus, is in the ground state, where its energy has the smallest value. In the ground state, the electron is most strongly bound to the nucleus and its energy is given by Equation 6.46. If we want to remove this electron from the atom, we must supply it with enough energy, E_∞, to at least balance out its ground state energy E_1:

$$E_\infty + E_1 = 0 \Rightarrow E_\infty = -E_1 = -(-E_0) = E_0 = 13.6\,\text{eV}. \quad (6.47)$$

The energy that is needed to remove the electron from the atom is called the **ionization energy**. The ionization energy E_∞ that is needed to remove the electron from the first Bohr orbit is called the **ionization limit of the hydrogen atom**. The ionization limit in Equation 6.47 that we obtain in Bohr's model agrees with experimental value.

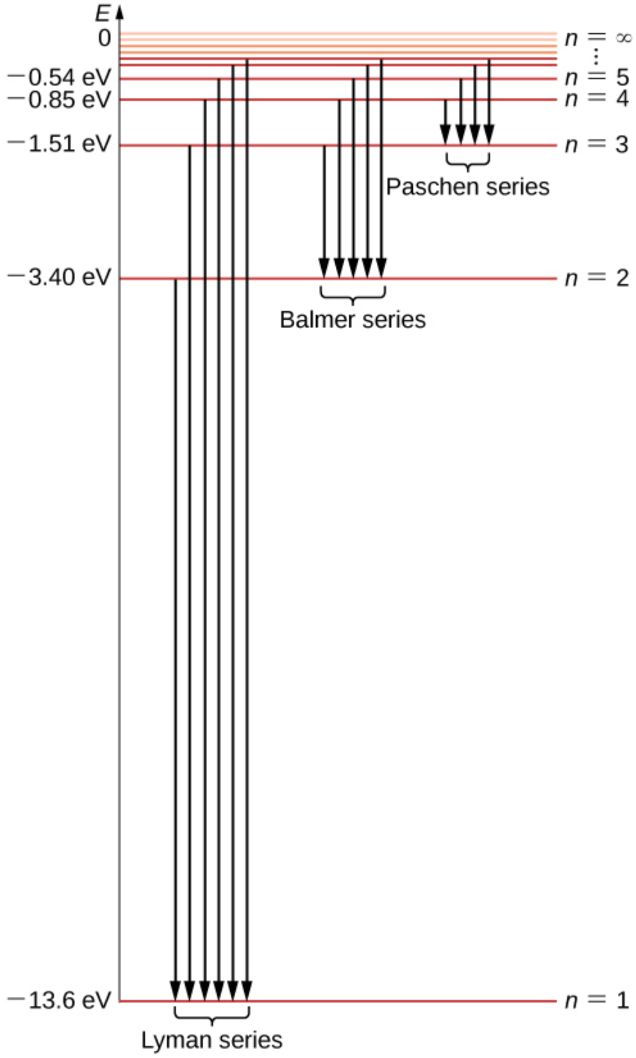


Figure 6.17 The energy spectrum of the hydrogen atom. Energy levels (horizontal lines) represent the bound states of an electron in the atom. There is only one ground state, $n = 1$, and infinite quantized excited states. The states are enumerated by the quantum number $n = 1,\ 2,\ 3,\ 4,\ \ldots.$ Vertical lines illustrate the allowed electron transitions between the states. Downward arrows illustrate transitions with an emission of a photon with a wavelength in the indicated spectral band.

Spectral Emission Lines of Hydrogen

To obtain the wavelengths of the emitted radiation when an electron makes a transition from the *n*th orbit to the *m*th orbit, we use the second of Bohr's quantization conditions and **Equation 6.45** for energies. The emission of energy from the atom can occur only when an electron makes a transition from an excited state to a lower-energy state. In the course of such a transition, the emitted photon carries away the difference of energies between the states involved in the transition. The transition cannot go in the other direction because the energy of a photon cannot be negative, which means that for emission we must have $E_n > E_m$ and $n > m.$ Therefore, the third of Bohr's postulates gives

$$hf = \left|E_n - E_m\right| = E_n - E_m = -E_0\frac{1}{n^2} + E_0\frac{1}{m^2} = E_0\left(\frac{1}{m^2} - \frac{1}{n^2}\right). \tag{6.48}$$

Now we express the photon's energy in terms of its wavelength, $hf = hc/\lambda,$ and divide both sides of **Equation 6.48** by $hc.$ The result is

$$\frac{1}{\lambda} = \frac{E_0}{hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right). \tag{6.49}$$

The value of the constant in this equation is

$$\frac{E_0}{hc} = \frac{13.6\,\text{eV}}{(4.136 \times 10^{-15}\,\text{eV}\cdot\text{s})(2.997 \times 10^8\,\text{m/s})} = 1.097 \times 10^7 \frac{1}{\text{m}}. \tag{6.50}$$

This value is exactly the Rydberg constant R_H in the Rydberg heuristic formula **Equation 6.32**. In fact, **Equation 6.49** is identical to the Rydberg formula, because for a given *m*, we have $n = m + 1,\ m + 2,\ ....$ In this way, the Bohr quantum model of the hydrogen atom allows us to derive the experimental Rydberg constant from first principles and to express it in terms of fundamental constants. Transitions between the allowed electron orbits are illustrated in **Figure 6.17**.

We can repeat the same steps that led to **Equation 6.49** to obtain the wavelength of the absorbed radiation; this again gives **Equation 6.49** but this time for the positions of absorption lines in the absorption spectrum of hydrogen. The only difference is that for absorption, the quantum number *m* is the index of the orbit occupied by the electron before the transition (lower-energy orbit) and the quantum number *n* is the index of the orbit to which the electron makes the transition (higher-energy orbit). The difference between the electron energies in these two orbits is the energy of the absorbed photon.

Example 6.10

Size and Ionization Energy of the Hydrogen Atom in an Excited State

If a hydrogen atom in the ground state absorbs a 93.7-nm photon, corresponding to a transition line in the Lyman series, how does this affect the atom's energy and size? How much energy is needed to ionize the atom when it is in this excited state? Give your answers in absolute units, and relative to the ground state.

Strategy

Before the absorption, the atom is in its ground state. This means that the electron transition takes place from the orbit $m = 1$ to some higher *n*th orbit. First, we must determine n for the absorbed wavelength $\lambda = 93.7\,\text{nm}$. Then, we can use **Equation 6.45** to find the energy E_n of the excited state and its ionization energy $E_{\infty,\,n}$, and use **Equation 6.40** to find the radius r_n of the atom in the excited state. To estimate *n*, we use **Equation 6.49**.

Solution

Substitute $m = 1$ and $\lambda = 93.7\,\text{nm}$ in **Equation 6.49** and solve for n. You should not expect to obtain a perfect integer answer because of rounding errors, but your answer will be close to an integer, and you can estimate *n* by taking the integral part of your answer:

$$\frac{1}{\lambda} = R_\text{H}\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow n = \frac{1}{\sqrt{1 - \frac{1}{\lambda R_\text{H}}}} = \frac{1}{\sqrt{1 - \frac{1}{(93.7 \times 10^{-9}\,\text{m})(1.097 \times 10^7\,\text{m}^{-1})}}} = 6.07 \Rightarrow n = 6.$$

The radius of the $n = 6$ orbit is

$$r_n = a_0 n^2 = a_0 6^2 = 36a_0 = 36(0.529 \times 10^{-10}\,\text{m}) = 19.04 \times 10^{-10}\,\text{m} \cong 19.0\,\text{Å}.$$

Thus, after absorbing the 93.7-nm photon, the size of the hydrogen atom in the excited $n = 6$ state is 36 times larger than before the absorption, when the atom was in the ground state. The energy of the fifth excited state ($n = 6$) is:

$$E_n = -\frac{E_0}{n^2} = -\frac{E_0}{6^2} = -\frac{E_0}{36} = -\frac{13.6\,\text{eV}}{36} \cong -0.378\,\text{eV}.$$

After absorbing the 93.7-nm photon, the energy of the hydrogen atom is larger than it was before the absorption. Ionization of the atom when it is in the fifth excited state ($n = 6$) requites 36 times less energy than is needed when the atom is in the ground state:

$$E_{\infty,\,6} = -E_6 = -(-0.378\,\text{eV}) = 0.378\,\text{eV}.$$

Significance

We can analyze any spectral line in the spectrum of hydrogen in the same way. Thus, the experimental measurements of spectral lines provide us with information about the atomic structure of the hydrogen atom.

6.10 Check Your Understanding When an electron in a hydrogen atom is in the first excited state, what prediction does the Bohr model give about its orbital speed and kinetic energy? What is the magnitude of its orbital angular momentum?

Bohr's model of the hydrogen atom also correctly predicts the spectra of some hydrogen-like ions. **Hydrogen-like ions** are atoms of elements with an atomic number Z larger than one ($Z = 1$ for hydrogen) but with all electrons removed except one. For example, an electrically neutral helium atom has an atomic number $Z = 2$. This means it has two electrons orbiting the nucleus with a charge of $q = +Ze$. When one of the orbiting electrons is removed from the helium atom (we say, when the helium atom is singly ionized), what remains is a hydrogen-like atomic structure where the remaining electron orbits the nucleus with a charge of $q = +Ze$. This type of situation is described by the Bohr model. Assuming that the charge of the nucleus is not $+e$ but $+Ze$, we can repeat all steps, beginning with **Equation 6.36**, to obtain the results for a hydrogen-like ion:

$$r_n = \frac{a_0}{Z} n^2 \tag{6.51}$$

where a_0 is the Bohr orbit of hydrogen, and

$$E_n = -Z^2 E_0 \frac{1}{n^2} \tag{6.52}$$

where E_0 is the ionization limit of a hydrogen atom. These equations are good approximations as long as the atomic number Z is not too large.

The Bohr model is important because it was the first model to postulate the quantization of electron orbits in atoms. Thus, it represents an early quantum theory that gave a start to developing modern quantum theory. It introduced the concept of a quantum number to describe atomic states. The limitation of the early quantum theory is that it cannot describe atoms in which the number of electrons orbiting the nucleus is larger than one. The Bohr model of hydrogen is a semi-classical model because it combines the classical concept of electron orbits with the new concept of quantization. The remarkable success of this model prompted many physicists to seek an explanation for why such a model should work at all, and to seek an understanding of the physics behind the postulates of early quantum theory. This search brought about the onset of an entirely new concept of "matter waves."

6.5 | De Broglie's Matter Waves

Learning Objectives

By the end of this section, you will be able to:

- Describe de Broglie's hypothesis of matter waves
- Explain how the de Broglie's hypothesis gives the rationale for the quantization of angular momentum in Bohr's quantum theory of the hydrogen atom
- Describe the Davisson–Germer experiment
- Interpret de Broglie's idea of matter waves and how they account for electron diffraction phenomena

Compton's formula established that an electromagnetic wave can behave like a particle of light when interacting with matter. In 1924, Louis de Broglie proposed a new speculative hypothesis that electrons and other particles of matter can behave like waves. Today, this idea is known as **de Broglie's hypothesis of matter waves**. In 1926, De Broglie's hypothesis, together with Bohr's early quantum theory, led to the development of a new theory of **wave quantum mechanics** to describe the physics of atoms and subatomic particles. Quantum mechanics has paved the way for new engineering inventions and technologies, such as the laser and magnetic resonance imaging (MRI). These new technologies drive discoveries in other sciences such as biology and chemistry.

According to de Broglie's hypothesis, massless photons as well as massive particles must satisfy one common set of relations that connect the energy E with the frequency f, and the linear momentum p with the wavelength λ. We have discussed these relations for photons in the context of Compton's effect. We are recalling them now in a more general context. Any particle that has energy and momentum is a **de Broglie wave** of frequency f and wavelength λ:

$$E = hf \tag{6.53}$$

$$\lambda = \frac{h}{p}. \tag{6.54}$$

Here, E and p are, respectively, the relativistic energy and the momentum of a particle. De Broglie's relations are usually expressed in terms of the wave vector $\vec{\mathbf{k}}$, $k = 2\pi/\lambda$, and the wave frequency $\omega = 2\pi f$, as we usually do for waves:

$$E = \hbar\omega \tag{6.55}$$

$$\vec{\mathbf{p}} = \hbar\vec{\mathbf{k}}. \tag{6.56}$$

Wave theory tells us that a wave carries its energy with the **group velocity**. For matter waves, this group velocity is the velocity u of the particle. Identifying the energy E and momentum p of a particle with its relativistic energy mc^2 and its relativistic momentum mu, respectively, it follows from de Broglie relations that matter waves satisfy the following relation:

$$\lambda f = \frac{\omega}{k} = \frac{E/\hbar}{p/\hbar} = \frac{E}{p} = \frac{mc^2}{mu} = \frac{c^2}{u} = \frac{c}{\beta} \tag{6.57}$$

where $\beta = u/c$. When a particle is massless we have $u = c$ and Equation 6.57 becomes $\lambda f = c$.

Example 6.11

How Long Are de Broglie Matter Waves?

Calculate the de Broglie wavelength of: (a) a 0.65-kg basketball thrown at a speed of 10 m/s, (b) a nonrelativistic electron with a kinetic energy of 1.0 eV, and (c) a relativistic electron with a kinetic energy of $108\,\text{keV}$.

Strategy

We use **Equation 6.57** to find the de Broglie wavelength. When the problem involves a nonrelativistic object moving with a nonrelativistic speed u, such as in (a) when $\beta = u/c \ll 1,$ we use nonrelativistic momentum p. When the nonrelativistic approximation cannot be used, such as in (c), we must use the relativistic momentum $p = mu = m_0\gamma u = E_0\gamma\beta,$ where the rest mass energy of a particle is $E_0 = m_0c^2$ and γ is the Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$. The total energy E of a particle is given by **Equation 6.53** and the kinetic energy is $K = E - E_0 = (\gamma - 1)E_0$. When the kinetic energy is known, we can invert **Equation 6.18** to find the momentum $p = \sqrt{(E^2 - E_0^2)/c^2} = \sqrt{K(K+2E_0)}/c$ and substitute in **Equation 6.57** to obtain

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K(K+2E_0)}}. \tag{6.58}$$

Depending on the problem at hand, in this equation we can use the following values for hc: $hc = (6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^{8}\,\text{m/s}) = 1.986 \times 10^{-25}\,\text{J}\cdot\text{m} = 1.241\,\text{eV}\cdot\mu\text{m}$

Solution

a. For the basketball, the kinetic energy is

$$K = m_0u^2/2 = (0.65\text{kg})(10\text{m/s})^2/2 = 32.5\text{J}$$

and the rest mass energy is

$$E_0 = m_0c^2 = (0.65\text{kg})(2.998 \times 10^8\,\text{m/s})^2 = 5.84 \times 10^{16}\,\text{J}.$$

We see that $K/(K+E_0) \ll 1$ and use $p = m_0u = (0.65\text{kg})(10\text{m/s}) = 6.5\,\text{J}\cdot\text{s/m}$:

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{6.5\text{J}\cdot\text{s/m}} = 1.02 \times 10^{-34}\,\text{m}.$$

b. For the nonrelativistic electron,

$$E_0 = m_0c^2 = (9.109 \times 10^{-31}\,\text{kg})(2.998 \times 10^8\,\text{m/s})^2 = 511\,\text{keV}$$

and when $K = 1.0\,\text{eV},$ we have $K/(K+E_0) = (1/512) \times 10^{-3} \ll 1,$ so we can use the nonrelativistic formula. However, it is simpler here to use **Equation 6.58**:

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K(K+2E_0)}} = \frac{1.241\text{ eV}\cdot\mu\text{m}}{\sqrt{(1.0\text{ eV})[1.0\text{ eV}+2(511\text{ keV})]}} = 1.23\text{ nm}.$$

If we use nonrelativistic momentum, we obtain the same result because 1 eV is much smaller than the rest mass of the electron.

c. For a fast electron with $K = 108\,\text{keV},$ relativistic effects cannot be neglected because its total energy is $E = K + E_0 = 108\,\text{keV} + 511\,\text{keV} = 619\,\text{keV}$ and $K/E = 108/619$ is not negligible:

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K(K+2E_0)}} = \frac{1.241\text{ eV}\cdot\text{µm}}{\sqrt{108\text{ keV}[108\text{ keV} + 2(511\text{ keV})]}} = 3.55\text{ pm}.$$

Significance

We see from these estimates that De Broglie's wavelengths of macroscopic objects such as a ball are immeasurably small. Therefore, even if they exist, they are not detectable and do not affect the motion of macroscopic objects.

6.11 Check Your Understanding What is de Broglie's wavelength of a nonrelativistic proton with a kinetic energy of 1.0 eV?

Using the concept of the electron matter wave, de Broglie provided a rationale for the quantization of the electron's angular momentum in the hydrogen atom, which was postulated in Bohr's quantum theory. The physical explanation for the first Bohr quantization condition comes naturally when we assume that an electron in a hydrogen atom behaves not like a particle but like a wave. To see it clearly, imagine a stretched guitar string that is clamped at both ends and vibrates in one of its normal modes. If the length of the string is l (**Figure 6.18**), the wavelengths of these vibrations cannot be arbitrary but must be such that an integer k number of half-wavelengths $\lambda/2$ fit exactly on the distance l between the ends. This is the condition $l = k\lambda/2$ for a standing wave on a string. Now suppose that instead of having the string clamped at the walls, we bend its length into a circle and fasten its ends to each other. This produces a circular string that vibrates in normal modes, satisfying the same standing-wave condition, but the number of half-wavelengths must now be an even number k, $k = 2n$, and the length l is now connected to the radius r_n of the circle. This means that the radii are not arbitrary but must satisfy the following standing-wave condition:

$$2\pi r_n = 2n\frac{\lambda}{2}. \tag{6.59}$$

If an electron in the nth Bohr orbit moves as a wave, by **Equation 6.59** its wavelength must be equal to $\lambda = 2\pi r_n/n$. Assuming that **Equation 6.58** is valid, the electron wave of this wavelength corresponds to the electron's linear momentum, $p = h/\lambda = nh/(2\pi r_n) = n\hbar/r_n$. In a circular orbit, therefore, the electron's angular momentum must be

$$L_n = r_n p = r_n \frac{n\hbar}{r_n} = n\hbar. \tag{6.60}$$

This equation is the first of Bohr's quantization conditions, given by **Equation 6.36**. Providing a physical explanation for Bohr's quantization condition is a convincing theoretical argument for the existence of matter waves.

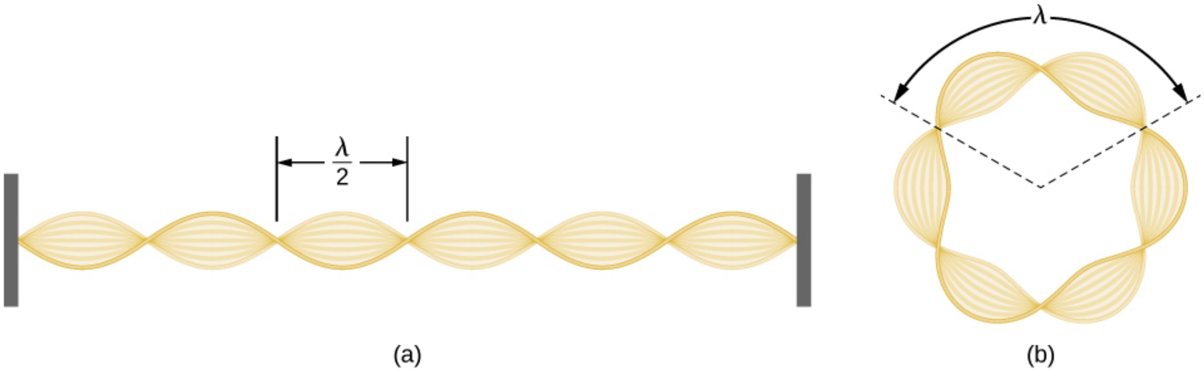


Figure 6.18 Standing-wave pattern: (a) a stretched string clamped at the walls; (b) an electron wave trapped in the third Bohr orbit in the hydrogen atom.

Example 6.12

The Electron Wave in the Ground State of Hydrogen

Find the de Broglie wavelength of an electron in the ground state of hydrogen.

Strategy

We combine the first quantization condition in **Equation 6.60** with **Equation 6.36** and use **Equation 6.38** for the first Bohr radius with $n = 1$.

Solution

When $n = 1$ and $r_n = a_0 = 0.529\ \text{Å}$, the Bohr quantization condition gives $a_0 p = 1 \cdot \hbar \Rightarrow p = \hbar / a_0$. The electron wavelength is:

$$\lambda = h/p = h/\hbar/a_0 = 2\pi a_0 = 2\pi(0.529\ \text{Å}) = 3.324\ \text{Å}.$$

Significance

We obtain the same result when we use **Equation 6.58** directly.

6.12 Check Your Understanding Find the de Broglie wavelength of an electron in the third excited state of hydrogen.

Experimental confirmation of matter waves came in 1927 when C. Davisson and L. Germer performed a series of electron-scattering experiments that clearly showed that electrons do behave like waves. Davisson and Germer did not set up their experiment to confirm de Broglie's hypothesis: The confirmation came as a byproduct of their routine experimental studies of metal surfaces under electron bombardment.

In the particular experiment that provided the very first evidence of electron waves (known today as the **Davisson–Germer experiment**), they studied a surface of nickel. Their nickel sample was specially prepared in a high-temperature oven to change its usual polycrystalline structure to a form in which large single-crystal domains occupy the volume. **Figure 6.19** shows the experimental setup. Thermal electrons are released from a heated element (usually made of tungsten) in the electron gun and accelerated through a potential difference ΔV, becoming a well-collimated beam of electrons produced by an electron gun. The kinetic energy K of the electrons is adjusted by selecting a value of the potential difference in the electron gun. This produces a beam of electrons with a set value of linear momentum, in accordance with the conservation of energy:

$$e\Delta V = K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2me\Delta V}. \tag{6.61}$$

The electron beam is incident on the nickel sample in the direction normal to its surface. At the surface, it scatters in various directions. The intensity of the beam scattered in a selected direction φ is measured by a highly sensitive detector. The detector's angular position with respect to the direction of the incident beam can be varied from $\varphi = 0°$ to $\varphi = 90°$. The entire setup is enclosed in a vacuum chamber to prevent electron collisions with air molecules, as such thermal collisions would change the electrons' kinetic energy and are not desirable.

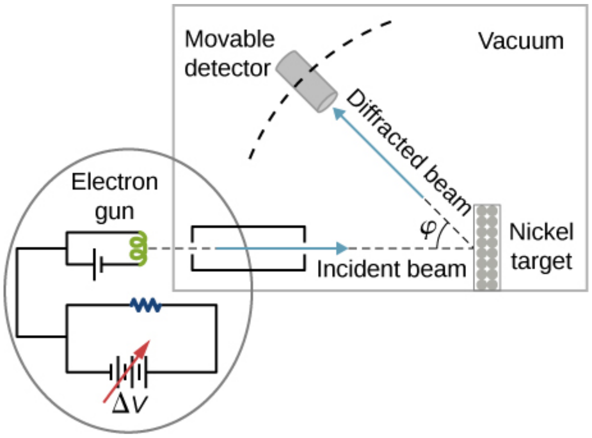


Figure 6.19 Schematics of the experimental setup of the Davisson–Germer diffraction experiment. A well-collimated beam of electrons is scattered off the nickel target. The kinetic energy of electrons in the incident beam is selected by adjusting a variable potential, $\Delta V,$ in the electron gun. Intensity of the scattered electron beam is measured for a range of scattering angles $\varphi,$ whereas the distance between the detector and the target does not change.

When the nickel target has a polycrystalline form with many randomly oriented microscopic crystals, the incident electrons scatter off its surface in various random directions. As a result, the intensity of the scattered electron beam is much the same in any direction, resembling a diffuse reflection of light from a porous surface. However, when the nickel target has a regular crystalline structure, the intensity of the scattered electron beam shows a clear maximum at a specific angle and the results show a clear diffraction pattern (see **Figure 6.20**). Similar diffraction patterns formed by X-rays scattered by various crystalline solids were studied in 1912 by father-and-son physicists William H. Bragg and William L. Bragg. The Bragg law in X-ray crystallography provides a connection between the wavelength λ of the radiation incident on a crystalline lattice, the lattice spacing, and the position of the interference maximum in the diffracted radiation (see **Diffraction**).

The lattice spacing of the Davisson–Germer target, determined with X-ray crystallography, was measured to be $a = 2.15\,\text{Å}.$ Unlike X-ray crystallography in which X-rays penetrate the sample, in the original Davisson–Germer experiment, only the surface atoms interact with the incident electron beam. For the surface diffraction, the maximum intensity of the reflected electron beam is observed for scattering angles that satisfy the condition $n\lambda = a\sin\varphi$ (see **Figure 6.21**). The first-order maximum (for $n = 1$) is measured at a scattering angle of $\varphi \approx 50°$ at $\Delta V \approx 54\text{V},$ which gives the wavelength of the incident radiation as $\lambda = (2.15\,\text{Å})\sin 50° = 1.64\,\text{Å}.$ On the other hand, a 54-V potential accelerates the incident electrons to kinetic energies of $K = 54\,\text{eV}.$ Their momentum, calculated from **Equation 6.61**, is $p = 2.478 \times 10^{-5}\,\text{eV} \cdot \text{s/m}.$ When we substitute this result in **Equation 6.58**, the de Broglie wavelength is obtained as

$$\lambda = \frac{h}{p} = \frac{4.136 \times 10^{-15}\,\text{eV} \cdot \text{s}}{2.478 \times 10^{-5}\,\text{eV} \cdot \text{s/m}} = 1.67\,\text{Å}. \tag{6.62}$$

The same result is obtained when we use $K = 54\,\text{eV}$ in **Equation 6.61**. The proximity of this theoretical result to the Davisson–Germer experimental value of $\lambda = 1.64\,\text{Å}$ is a convincing argument for the existence of de Broglie matter waves.

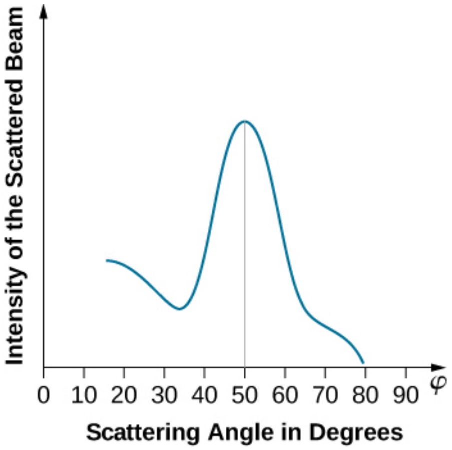
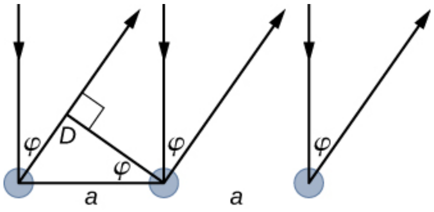


Figure 6.20 The experimental results of electron diffraction on a nickel target for the accelerating potential in the electron gun of about $\Delta V = 54\text{V}:$ The intensity maximum is registered at the scattering angle of about $\varphi = 50°.$



$$D = a \sin \varphi$$
$$D = n\lambda \quad n = 1, 2, 3, \ldots$$
$$n\lambda = a \sin \varphi$$

Figure 6.21 In the surface diffraction of a monochromatic electromagnetic wave on a crystalline lattice structure, the in-phase incident beams are reflected from atoms on the surface. A ray reflected from the left atom travels an additional distance $D = a\sin\varphi$ to the detector, where *a* is the lattice spacing. The reflected beams remain in-phase when *D* is an integer multiple of their wavelength $\lambda.$ The intensity of the reflected waves has pronounced maxima for angles φ satisfying $n\lambda = a\sin\varphi.$

Diffraction lines measured with low-energy electrons, such as those used in the Davisson–Germer experiment, are quite broad (see **Figure 6.20**) because the incident electrons are scattered only from the surface. The resolution of diffraction images greatly improves when a higher-energy electron beam passes through a thin metal foil. This occurs because the diffraction image is created by scattering off many crystalline planes inside the volume, and the maxima produced in scattering at Bragg angles are sharp (see **Figure 6.22**).

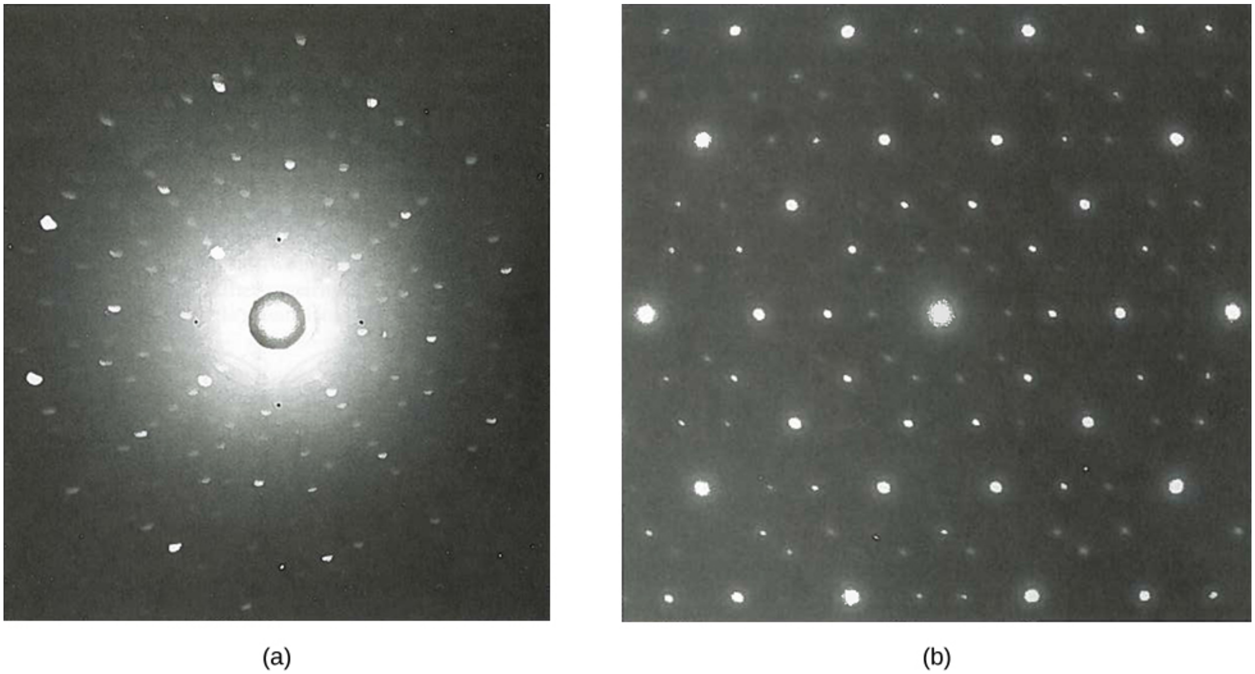


Figure 6.22 Diffraction patterns obtained in scattering on a crystalline solid: (a) with X-rays, and (b) with electrons. The observed pattern reflects the symmetry of the crystalline structure of the sample.

Since the work of Davisson and Germer, de Broglie's hypothesis has been extensively tested with various experimental techniques, and the existence of de Broglie waves has been confirmed for numerous elementary particles. Neutrons have been used in scattering experiments to determine crystalline structures of solids from interference patterns formed by neutron matter waves. The neutron has zero charge and its mass is comparable with the mass of a positively charged proton. Both neutrons and protons can be seen as matter waves. Therefore, the property of being a matter wave is not specific to electrically charged particles but is true of all particles in motion. Matter waves of molecules as large as carbon C_{60} have been measured. All physical objects, small or large, have an associated matter wave as long as they remain in motion. The universal character of de Broglie matter waves is firmly established.

Example 6.13

Neutron Scattering

Suppose that a neutron beam is used in a diffraction experiment on a typical crystalline solid. Estimate the kinetic energy of a neutron (in eV) in the neutron beam and compare it with kinetic energy of an ideal gas in equilibrium at room temperature.

Strategy

We assume that a typical crystal spacing a is of the order of 1.0 Å. To observe a diffraction pattern on such a lattice, the neutron wavelength λ must be on the same order of magnitude as the lattice spacing. We use **Equation 6.61** to find the momentum p and kinetic energy K. To compare this energy with the energy E_T of ideal gas in equilibrium at room temperature $T = 300\text{K}$, we use the relation $K = \frac{3}{2}k_B T$, where $k_B = 8.62 \times 10^{-5}\,\text{eV/K}$ is the Boltzmann constant.

Solution

We evaluate pc to compare it with the neutron's rest mass energy $E_0 = 940\,\text{MeV}$:

$$p = \frac{h}{\lambda} \Rightarrow pc = \frac{hc}{\lambda} = \frac{1.241 \times 10^{-6}\,\text{eV} \cdot \text{m}}{10^{-10}\,\text{m}} = 12.41\,\text{keV}.$$

We see that $p^2c^2 \ll E_0^2$ so $K \ll E_0$ and we can use the nonrelativistic kinetic energy:

$$K = \frac{p^2}{2m_n} = \frac{h^2}{2\lambda^2 m_n} = \frac{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})^2}{(2 \times 10^{-20}\,\text{m}^2)(1.66 \times 10^{-27}\,\text{kg})} = 1.32 \times 10^{-20}\,\text{J} = 82.7\,\text{meV}.$$

Kinetic energy of ideal gas in equilibrium at 300 K is:

$$K_T = \frac{3}{2}k_B T = \frac{3}{2}(8.62 \times 10^{-5}\,\text{eV/K})(300\text{K}) = 38.8\,\text{MeV}.$$

We see that these energies are of the same order of magnitude.

Significance

Neutrons with energies in this range, which is typical for an ideal gas at room temperature, are called "thermal neutrons."

Example 6.14

Wavelength of a Relativistic Proton

In a supercollider at CERN, protons can be accelerated to velocities of 0.75*c*. What are their de Broglie wavelengths at this speed? What are their kinetic energies?

Strategy

The rest mass energy of a proton is $E_0 = m_0 c^2 = (1.672 \times 10^{-27}\,\text{kg})(2.998 \times 10^8\,\text{m/s})^2 = 938\,\text{MeV}$. When the proton's velocity is known, we have $\beta = 0.75$ and $\beta\gamma = 0.75/\sqrt{1 - 0.75^2} = 1.714$. We obtain the wavelength λ and kinetic energy K from relativistic relations.

Solution

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\beta\gamma E_0} = \frac{1.241\,\text{eV}\cdot\mu\text{m}}{1.714(938\,\text{MeV})} = 0.77\,\text{fm}$$

$$K = E_0(\gamma - 1) = 938\,\text{MeV}(1/\sqrt{1 - 0.75^2} - 1) = 480.1\,\text{MeV}$$

Significance

Notice that because a proton is 1835 times more massive than an electron, if this experiment were performed with electrons, a simple rescaling of these results would give us the electron's wavelength of $(1835)0.77\text{fm} = 1.4\,\text{pm}$ and its kinetic energy of $480.1\,\text{MeV}/1835 = 261.6\,\text{keV}$.

6.13 Check Your Understanding Find the de Broglie wavelength and kinetic energy of a free electron that travels at a speed of 0.75*c*.

6.6 | Wave-Particle Duality

Learning Objectives

By the end of this section, you will be able to:

- Identify phenomena in which electromagnetic waves behave like a beam of photons and particles behave like waves
- Describe the physics principles behind electron microscopy
- Summarize the evolution of scientific thought that led to the development of quantum mechanics

The energy of radiation detected by a radio-signal receiving antenna comes as the energy of an electromagnetic wave. The same energy of radiation detected by a photocurrent in the photoelectric effect comes as the energy of individual photon particles. Therefore, the question arises about the nature of electromagnetic radiation: Is a photon a wave or is it a particle? Similar questions may be asked about other known forms of energy. For example, an electron that forms part of an electric current in a circuit behaves like a particle moving in unison with other electrons inside the conductor. The same electron behaves as a wave when it passes through a solid crystalline structure and forms a diffraction image. Is an electron a wave or is it a particle? The same question can be extended to all particles of matter—elementary particles, as well as compound molecules—asking about their true physical nature. At our present state of knowledge, such questions about the true nature of things do not have conclusive answers. All we can say is that **wave-particle duality** exists in nature: Under some experimental conditions, a particle appears to act as a particle, and under different experimental conditions, a particle appears to act a wave. Conversely, under some physical circumstances electromagnetic radiation acts as a wave, and under other physical circumstances, radiation acts as a beam of photons.

This dualistic interpretation is not a new physics concept brought about by specific discoveries in the twentieth century. It was already present in a debate between Isaac Newton and Christiaan Huygens about the nature of light, beginning in the year 1670. According to Newton, a beam of light is a collection of corpuscles of light. According to Huygens, light is a wave. The corpuscular hypothesis failed in 1803, when Thomas Young announced his **double-slit interference experiment** with light (see Figure 6.23), which firmly established light as a wave. In James Clerk Maxwell's theory of electromagnetism (completed by the year 1873), light is an electromagnetic wave. Maxwell's classical view of radiation as an electromagnetic wave is still valid today; however, it is unable to explain blackbody radiation and the photoelectric effect, where light acts as a beam of photons.

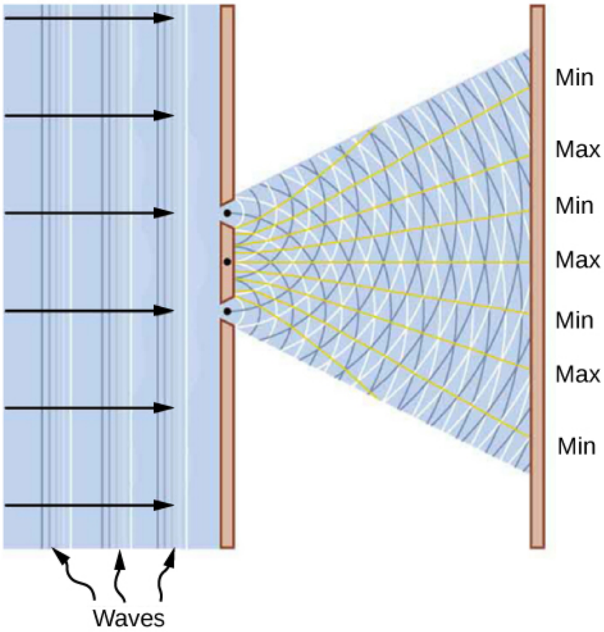


Figure 6.23 Young's double-slit experiment explains the interference of light by making an analogy with the interference of water waves. Two waves are generated at the positions of two slits in an opaque screen. The waves have the same wavelengths. They travel from their origins at the slits to the viewing screen placed to the right of the slits. The waves meet on the viewing screen. At the positions marked "Max" on the screen, the meeting waves are in-phase and the combined wave amplitude is enhanced. At positions marked "Min," the combined wave amplitude is zero. For light, this mechanism creates a bright-and-dark fringe pattern on the viewing screen.

A similar dichotomy existed in the interpretation of electricity. From Benjamin Franklin's observations of electricity in 1751 until J.J. Thomson's discovery of the electron in 1897, electric current was seen as a flow in a continuous electric medium. Within this theory of electric fluid, the present theory of electric circuits was developed, and electromagnetism and electromagnetic induction were discovered. Thomson's experiment showed that the unit of negative electric charge (an electron) can travel in a vacuum without any medium to carry the charge around, as in electric circuits. This discovery changed the way in which electricity is understood today and gave the electron its particle status. In Bohr's early quantum theory of the hydrogen atom, both the electron and the proton are particles of matter. Likewise, in the Compton scattering of X-rays on electrons, the electron is a particle. On the other hand, in electron-scattering experiments on crystalline structures, the electron behaves as a wave.

A skeptic may raise a question that perhaps an electron might always be nothing more than a particle, and that the diffraction images obtained in electron-scattering experiments might be explained within some macroscopic model of a crystal and a macroscopic model of electrons coming at it like a rain of ping-pong balls. As a matter of fact, to investigate this question, we do not need a complex model of a crystal but just a couple of simple slits in a screen that is opaque to electrons. In other words, to gather convincing evidence about the nature of an electron, we need to repeat the Young double-slit experiment with electrons. If the electron is a wave, we should observe the formation of interference patterns typical for waves, such as those described in **Figure 6.23**, even when electrons come through the slits one by one. However, if the electron is a not a wave but a particle, the interference fringes will not be formed.

The very first double-slit experiment with a beam of electrons, performed by Claus Jönsson in Germany in 1961, demonstrated that a beam of electrons indeed forms an interference pattern, which means that electrons collectively behave as a wave. The first double-slit experiments with *single* electrons passing through the slits one-by-one were performed by Giulio Pozzi in 1974 in Italy and by Akira Tonomura in 1989 in Japan. They show that interference fringes are formed gradually, even when electrons pass through the slits individually. This demonstrates conclusively that electron-diffraction images are formed because of the wave nature of electrons. The results seen in double-slit experiments with electrons are illustrated by the images of the interference pattern in **Figure 6.24**.

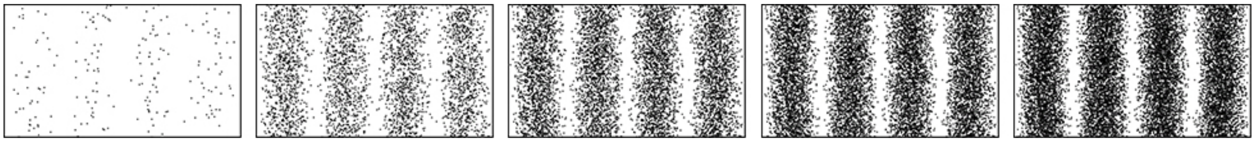


Figure 6.24 Computer-simulated interference fringes seen in the Young double-slit experiment with electrons. One pattern is gradually formed on the screen, regardless of whether the electrons come through the slits as a beam or individually one-by-one.

Example 6.15

Double-Slit Experiment with Electrons

In one experimental setup for studying interference patterns of electron waves, two slits are created in a gold-coated silicon membrane. Each slit is 62-nm wide and $4\text{-}\mu\text{m}$ long, and the separation between the slits is 272 nm.

The electron beam is created in an electron gun by heating a tungsten element and by accelerating the electrons across a 600-V potential. The beam is subsequently collimated using electromagnetic lenses, and the collimated beam of electrons is sent through the slits. Find the angular position of the first-order bright fringe on the viewing screen.

Strategy

Recall that the angular position θ of the nth order bright fringe that is formed in Young's two-slit interference pattern (discussed in a previous chapter) is related to the separation, d, between the slits and to the wavelength, $\lambda,$ of the incident light by the equation $d\sin\theta = n\lambda,$ where $n = 0, \ \pm 1, \ \pm 2, \ ....$ The separation is given and is equal to $d = 272 \text{ nm}.$ For the first-order fringe, we take $n = 1.$ The only thing we now need is the wavelength of the incident electron wave.

Since the electron has been accelerated from rest across a potential difference of $\Delta V = 600\text{V},$ its kinetic energy is $K = e\Delta V = 600 \text{ eV}.$ The rest-mass energy of the electron is $E_0 = 511 \text{ keV}.$

We compute its de Broglie wavelength as that of a nonrelativistic electron because its kinetic energy K is much smaller than its rest energy $E_0,$ $K \ll E_0.$

Solution

The electron's wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}} = \frac{h}{\sqrt{2E_0/c^2 K}} = \frac{hc}{\sqrt{2E_0 K}} = \frac{1.241 \times 10^{-6} \text{eV} \cdot \text{m}}{\sqrt{2(511 \text{ keV})(600 \text{ eV})}} = 0.050 \text{ nm}.$$

This λ is used to obtain the position of the first bright fringe:

$$\sin\theta = \frac{1 \cdot \lambda}{d} = \frac{0.050 \text{ nm}}{272 \text{ nm}} = 0.000184 \Rightarrow \theta = 0.010°.$$

Significance

Notice that this is also the angular resolution between two consecutive bright fringes up to about $n = 1000.$ For example, between the zero-order fringe and the first-order fringe, between the first-order fringe and the second-order fringe, and so on.

6.14 Check Your Understanding For the situation described in **Example 6.15**, find the angular position of the fifth-order bright fringe on the viewing screen.

The wave-particle dual nature of matter particles and of radiation is a declaration of our inability to describe physical reality within one unified classical theory because separately neither a classical particle approach nor a classical wave approach can fully explain the observed phenomena. This limitation of the classical approach was realized by the year 1928, and a foundation for a new statistical theory, called quantum mechanics, was put in place by Bohr, Edwin Schrödinger, Werner Heisenberg, and Paul Dirac. Quantum mechanics takes de Broglie's idea of matter waves to be the fundamental property of all particles and gives it a statistical interpretation. According to this interpretation, a wave that is associated with a particle carries information about the probable positions of the particle and about its other properties. A single particle is seen as a moving *wave packet* such as the one shown in **Figure 6.25**. We can intuitively sense from this example that if a particle is a wave packet, we will not be able to measure its exact position in the same sense as we cannot pinpoint a location of a wave packet in a vibrating guitar string. The uncertainty, $\Delta x,$ in measuring the particle's position is connected to the uncertainty, $\Delta p,$ in the simultaneous measuring of its linear momentum by Heisenberg's uncertainty principle:

$$\Delta x \Delta p \geq \frac{1}{2}\hbar. \tag{6.63}$$

Heisenberg's principle expresses the law of nature that, at the quantum level, our perception is limited. For example, if we know the exact position of a body (which means that $\Delta x = 0$ in **Equation 6.63**) at the same time we cannot know its momentum, because then the uncertainty in its momentum becomes infinite (because $\Delta p \geq 0.5\hbar / \Delta x$ in **Equation 6.63**). The **Heisenberg uncertainty principle** sets the limit on the precision of *simultaneous* measurements of position and momentum of a particle; it shows that the best precision we can obtain is when we have an equals sign ($=$) in **Equation 6.63**, and we cannot do better than that, even with the best instruments of the future. Heisenberg's principle is a consequence of the wave nature of particles.

Figure 6.25 In this graphic, a particle is shown as a wave packet and its position does not have an exact value.

We routinely use many electronic devices that exploit wave-particle duality without even realizing the sophistication of the physics underlying their operation. One example of a technology based on the particle properties of photons and electrons is a charge-coupled device, which is used for light detection in any instrumentation where high-quality digital data are required, such as in digital cameras or in medical sensors. An example in which the wave properties of electrons is exploited is an electron microscope.

In 1931, physicist Ernst Ruska—building on the idea that magnetic fields can direct an electron beam just as lenses can direct a beam of light in an optical microscope—developed the first prototype of the electron microscope. This development originated the field of **electron microscopy**. In the transmission electron microscope (TEM), shown in **Figure 6.26**, electrons are produced by a hot tungsten element and accelerated by a potential difference in an electron gun, which gives them up to 400 keV in kinetic energy. After leaving the electron gun, the electron beam is focused by electromagnetic lenses (a system of condensing lenses) and transmitted through a specimen sample to be viewed. The image of the sample is reconstructed from the transmitted electron beam. The magnified image may be viewed either directly on a fluorescent screen or indirectly by sending it, for example, to a digital camera or a computer monitor. The entire setup consisting of the electron gun, the lenses, the specimen, and the fluorescent screen are enclosed in a vacuum chamber to prevent the energy loss from the beam. Resolution of the TEM is limited only by spherical aberration (discussed in a previous chapter). Modern high-resolution models of a TEM can have resolving power greater than 0.5 Å and magnifications higher than 50 million times. For comparison, the best resolving power obtained with light microscopy is currently about 97 nm. A limitation of the TEM is that the samples must be about 100-nm thick and biological samples require a special preparation involving chemical "fixing" to stabilize them for ultrathin slicing.

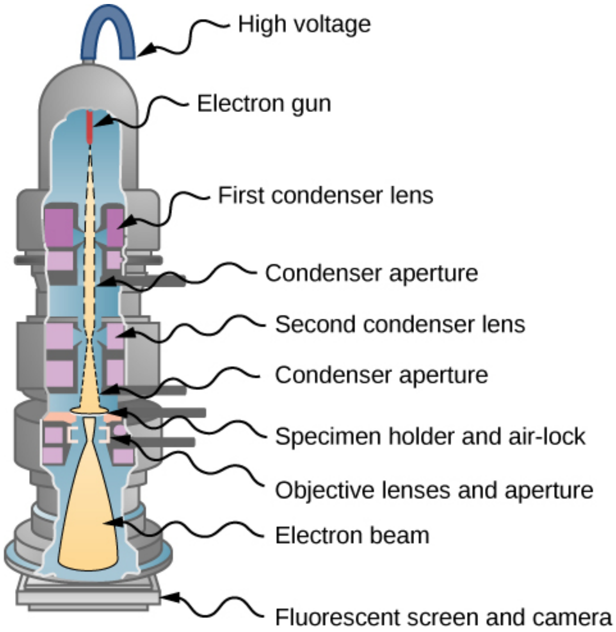


Figure 6.26 TEM: An electron beam produced by an electron gun is collimated by condenser lenses and passes through a specimen. The transmitted electrons are projected on a screen and the image is sent to a camera. (credit: modification of work by Dr. Graham Beards)

Such limitations do not appear in the scanning electron microscope (SEM), which was invented by Manfred von Ardenne in 1937. In an SEM, a typical energy of the electron beam is up to 40 keV and the beam is not transmitted through a sample but is scattered off its surface. Surface topography of the sample is reconstructed by analyzing back-scattered electrons, transmitted electrons, and the emitted radiation produced by electrons interacting with atoms in the sample. The resolving power of an SEM is better than 1 nm, and the magnification can be more than 250 times better than that obtained with a light microscope. The samples scanned by an SEM can be as large as several centimeters but they must be specially prepared, depending on electrical properties of the sample.

High magnifications of the TEM and SEM allow us to see individual molecules. High resolving powers of the TEM and SEM allow us to see fine details, such as those shown in the SEM micrograph of pollen at the beginning of this chapter (**Figure 6.1**).

Example 6.16

Resolving Power of an Electron Microscope

If a 1.0-pm electron beam of a TEM passes through a $2.0\text{-}\mu\text{m}$ circular opening, what is the angle between the two just-resolvable point sources for this microscope?

Solution

We can directly use a formula for the resolving power, $\Delta\theta,$ of a microscope (discussed in a previous chapter) when the wavelength of the incident radiation is $\lambda = 1.0\,\text{pm}$ and the diameter of the aperture is $D = 2.0\mu\text{m}$:

$$\Delta\theta = 1.22\frac{\lambda}{D} = 1.22\frac{1.0\,\text{pm}}{2.0\mu\text{m}} = 6.10\times 10^{-7}\,\text{rad} = 3.50\times 10^{-5}\,\text{degree}.$$

Significance

Note that if we used a conventional microscope with a 400-nm light, the resolving power would be only $14°$, which means that all of the fine details in the image would be blurred.

6.15 Check Your Understanding Suppose that the diameter of the aperture in **Example 6.16** is halved. How does it affect the resolving power?

CHAPTER 6 REVIEW

KEY TERMS

absorber any object that absorbs radiation

absorption spectrum wavelengths of absorbed radiation by atoms and molecules

Balmer formula describes the emission spectrum of a hydrogen atom in the visible-light range

Balmer series spectral lines corresponding to electron transitions to/from the $n = 2$ state of the hydrogen atom, described by the Balmer formula

blackbody perfect absorber/emitter

blackbody radiation radiation emitted by a blackbody

Bohr radius of hydrogen radius of the first Bohr’s orbit

Bohr’s model of the hydrogen atom first quantum model to explain emission spectra of hydrogen

Brackett series spectral lines corresponding to electron transitions to/from the $n = 4$ state

Compton effect the change in wavelength when an X-ray is scattered by its interaction with some materials

Compton shift difference between the wavelengths of the incident X-ray and the scattered X-ray

Compton wavelength physical constant with the value $\lambda_c = 2.43\text{ pm}$

cut-off frequency frequency of incident light below which the photoelectric effect does not occur

cut-off wavelength wavelength of incident light that corresponds to cut-off frequency

Davisson–Germer experiment historically first electron-diffraction experiment that revealed electron waves

de Broglie wave matter wave associated with any object that has mass and momentum

de Broglie’s hypothesis of matter waves particles of matter can behave like waves

double-slit interference experiment Young’s double-slit experiment, which shows the interference of waves

electron microscopy microscopy that uses electron waves to “see” fine details of nano-size objects

emission spectrum wavelengths of emitted radiation by atoms and molecules

emitter any object that emits radiation

energy of a photon quantum of radiant energy, depends only on a photon’s frequency

energy spectrum of hydrogen set of allowed discrete energies of an electron in a hydrogen atom

excited energy states of the H atom energy state other than the ground state

Fraunhofer lines dark absorption lines in the continuum solar emission spectrum

ground state energy of the hydrogen atom energy of an electron in the first Bohr orbit of the hydrogen atom

group velocity velocity of a wave, energy travels with the group velocity

Heisenberg uncertainty principle sets the limits on precision in simultaneous measurements of momentum and position of a particle

Humphreys series spectral lines corresponding to electron transitions to/from the $n = 6$ state

hydrogen-like atom ionized atom with one electron remaining and nucleus with charge $+Ze$

inelastic scattering scattering effect where kinetic energy is not conserved but the total energy is conserved

ionization energy energy needed to remove an electron from an atom

ionization limit of the hydrogen atom ionization energy needed to remove an electron from the first Bohr orbit

Lyman series spectral lines corresponding to electron transitions to/from the ground state

nuclear model of the atom heavy positively charged nucleus at the center is surrounded by electrons, proposed by Rutherford

Paschen series spectral lines corresponding to electron transitions to/from the $n = 3$ state

Pfund series spectral lines corresponding to electron transitions to/from the $n = 5$ state

photocurrent in a circuit, current that flows when a photoelectrode is illuminated

photoelectric effect emission of electrons from a metal surface exposed to electromagnetic radiation of the proper frequency

photoelectrode in a circuit, an electrode that emits photoelectrons

photoelectron electron emitted from a metal surface in the presence of incident radiation

photon particle of light

Planck’s hypothesis of energy quanta energy exchanges between the radiation and the walls take place only in the form of discrete energy quanta

postulates of Bohr’s model three assumptions that set a frame for Bohr’s model

power intensity energy that passes through a unit surface per unit time

propagation vector vector with magnitude $2\pi/\lambda$ that has the direction of the photon’s linear momentum

quantized energies discrete energies; not continuous

quantum number index that enumerates energy levels

quantum phenomenon in interaction with matter, photon transfers either all its energy or nothing

quantum state of a Planck’s oscillator any mode of vibration of Planck’s oscillator, enumerated by quantum number

reduced Planck’s constant Planck’s constant divided by 2π

Rutherford’s gold foil experiment first experiment to demonstrate the existence of the atomic nucleus

Rydberg constant for hydrogen physical constant in the Balmer formula

Rydberg formula experimentally found positions of spectral lines of hydrogen atom

scattering angle angle between the direction of the scattered beam and the direction of the incident beam

Stefan–Boltzmann constant physical constant in Stefan’s law

stopping potential in a circuit, potential difference that stops photocurrent

wave number magnitude of the propagation vector

wave quantum mechanics theory that explains the physics of atoms and subatomic particles

wave-particle duality particles can behave as waves and radiation can behave as particles

work function energy needed to detach photoelectron from the metal surface

α-particle doubly ionized helium atom

α-ray beam of α-particles (alpha-particles)

β-ray beam of electrons

γ-ray beam of highly energetic photons

KEY EQUATIONS

Wien’s displacement law	$\lambda_{\text{max}} T = 2.898 \times 10^{-3}\,\text{m} \cdot \text{K}$
Stefan’s law	$P(T) = \sigma A T^4$

Planck’s constant	$h = 6.626 \times 10^{-34}\,\text{J} \cdot \text{s} = 4.136 \times 10^{-15}\,\text{eV} \cdot \text{s}$
Energy quantum of radiation	$\Delta E = hf$
Planck’s blackbody radiation law	$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda k_B T} - 1}$
Maximum kinetic energy of a photoelectron	$K_{\text{max}} = e\Delta V_s$
Energy of a photon	$E_f = hf$
Energy balance for photoelectron	$K_{\text{max}} = hf - \phi$
Cut-off frequency	$f_c = \frac{\phi}{h}$
Relativistic invariant energy equation	$E^2 = p^2c^2 + m_0^2c^4$
Energy-momentum relation for photon	$p_f = \frac{E_f}{c}$
Energy of a photon	$E_f = hf = \frac{hc}{\lambda}$
Magnitude of photon’s momentum	$p_f = \frac{h}{\lambda}$
Photon’s linear momentum vector	$\vec{p}_f = \hbar \vec{k}$
The Compton wavelength of an electron	$\lambda_c = \frac{h}{m_0 c} = 0.00243\,\text{nm}$
The Compton shift	$\Delta\lambda = \lambda_c(1 - \cos\theta)$
The Balmer formula	$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$
The Rydberg formula	$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), n_i = n_f + 1, n_f + 2, \ldots$
Bohr’s first quantization condition	$L_n = n\hbar, n = 1, 2, \ldots$
Bohr’s second quantization condition	$hf = \lvert E_n - E_m \rvert$
Bohr’s radius of hydrogen	$a_0 = 4\pi\varepsilon_0\frac{\hbar^2}{m_e e^2} = 0.529\,\text{Å}$
Bohr’s radius of the *n*th orbit	$r_n = a_0 n^2$
Ground-state energy value, ionization limit	$E_0 = \frac{1}{8\varepsilon_0^2}\frac{m_e e^4}{h^2} = 13.6\,\text{eV}$
Electron’s energy in the *n*th orbit	$E_n = -E_0\frac{1}{n^2}$
Ground state energy of hydrogen	$E_1 = -E_0 = -13.6\,\text{eV}$

The *n*th orbit of hydrogen-like ion	$r_n = \frac{a_0}{Z}n^2$
The *n*th energy of hydrogen-like ion	$E_n = -Z^2 E_0 \frac{1}{n^2}$
Energy of a matter wave	$E = hf$
The de Broglie wavelength	$\lambda = \frac{h}{p}$
The frequency-wavelength relation for matter waves	$\lambda f = \frac{c}{\beta}$
Heisenberg's uncertainty principle	$\Delta x \Delta p \geq \frac{1}{2}\hbar$

SUMMARY

6.1 Blackbody Radiation

- All bodies radiate energy. The amount of radiation a body emits depends on its temperature. The experimental Wien's displacement law states that the hotter the body, the shorter the wavelength corresponding to the emission peak in the radiation curve. The experimental Stefan's law states that the total power of radiation emitted across the entire spectrum of wavelengths at a given temperature is proportional to the fourth power of the Kelvin temperature of the radiating body.
- Absorption and emission of radiation are studied within the model of a blackbody. In the classical approach, the exchange of energy between radiation and cavity walls is continuous. The classical approach does not explain the blackbody radiation curve.
- To explain the blackbody radiation curve, Planck assumed that the exchange of energy between radiation and cavity walls takes place only in discrete quanta of energy. Planck's hypothesis of energy quanta led to the theoretical Planck's radiation law, which agrees with the experimental blackbody radiation curve; it also explains Wien's and Stefan's laws.

6.2 Photoelectric Effect

- The photoelectric effect occurs when photoelectrons are ejected from a metal surface in response to monochromatic radiation incident on the surface. It has three characteristics: (1) it is instantaneous, (2) it occurs only when the radiation is above a cut-off frequency, and (3) kinetic energies of photoelectrons at the surface do not depend of the intensity of radiation. The photoelectric effect cannot be explained by classical theory.
- We can explain the photoelectric effect by assuming that radiation consists of photons (particles of light). Each photon carries a quantum of energy. The energy of a photon depends only on its frequency, which is the frequency of the radiation. At the surface, the entire energy of a photon is transferred to one photoelectron.
- The maximum kinetic energy of a photoelectron at the metal surface is the difference between the energy of the incident photon and the work function of the metal. The work function is the binding energy of electrons to the metal surface. Each metal has its own characteristic work function.

6.3 The Compton Effect

- In the Compton effect, X-rays scattered off some materials have different wavelengths than the wavelength of the incident X-rays. This phenomenon does not have a classical explanation.
- The Compton effect is explained by assuming that radiation consists of photons that collide with weakly bound electrons in the target material. Both electron and photon are treated as relativistic particles. Conservation laws of the total energy and of momentum are obeyed in collisions.
- Treating the photon as a particle with momentum that can be transferred to an electron leads to a theoretical Compton shift that agrees with the wavelength shift measured in the experiment. This provides evidence that radiation consists of photons.

- Compton scattering is an inelastic scattering, in which scattered radiation has a longer wavelength than that of incident radiation.

6.4 Bohr's Model of the Hydrogen Atom

- Positions of absorption and emission lines in the spectrum of atomic hydrogen are given by the experimental Rydberg formula. Classical physics cannot explain the spectrum of atomic hydrogen.
- The Bohr model of hydrogen was the first model of atomic structure to correctly explain the radiation spectra of atomic hydrogen. It was preceded by the Rutherford nuclear model of the atom. In Rutherford's model, an atom consists of a positively charged point-like nucleus that contains almost the entire mass of the atom and of negative electrons that are located far away from the nucleus.
- Bohr's model of the hydrogen atom is based on three postulates: (1) an electron moves around the nucleus in a circular orbit, (2) an electron's angular momentum in the orbit is quantized, and (3) the change in an electron's energy as it makes a quantum jump from one orbit to another is always accompanied by the emission or absorption of a photon. Bohr's model is semi-classical because it combines the classical concept of electron orbit (postulate 1) with the new concept of quantization (postulates 2 and 3).
- Bohr's model of the hydrogen atom explains the emission and absorption spectra of atomic hydrogen and hydrogen-like ions with low atomic numbers. It was the first model to introduce the concept of a quantum number to describe atomic states and to postulate quantization of electron orbits in the atom. Bohr's model is an important step in the development of quantum mechanics, which deals with many-electron atoms.

6.5 De Broglie's Matter Waves

- De Broglie's hypothesis of matter waves postulates that any particle of matter that has linear momentum is also a wave. The wavelength of a matter wave associated with a particle is inversely proportional to the magnitude of the particle's linear momentum. The speed of the matter wave is the speed of the particle.
- De Broglie's concept of the electron matter wave provides a rationale for the quantization of the electron's angular momentum in Bohr's model of the hydrogen atom.
- In the Davisson–Germer experiment, electrons are scattered off a crystalline nickel surface. Diffraction patterns of electron matter waves are observed. They are the evidence for the existence of matter waves. Matter waves are observed in diffraction experiments with various particles.

6.6 Wave-Particle Duality

- Wave-particle duality exists in nature: Under some experimental conditions, a particle acts as a particle; under other experimental conditions, a particle acts as a wave. Conversely, under some physical circumstances, electromagnetic radiation acts as a wave, and under other physical circumstances, radiation acts as a beam of photons.
- Modern-era double-slit experiments with electrons demonstrated conclusively that electron-diffraction images are formed because of the wave nature of electrons.
- The wave-particle dual nature of particles and of radiation has no classical explanation.
- Quantum theory takes the wave property to be the fundamental property of all particles. A particle is seen as a moving wave packet. The wave nature of particles imposes a limitation on the simultaneous measurement of the particle's position and momentum. Heisenberg's uncertainty principle sets the limits on precision in such simultaneous measurements.
- Wave-particle duality is exploited in many devices, such as charge-couple devices (used in digital cameras) or in the electron microscopy of the scanning electron microscope (SEM) and the transmission electron microscope (TEM).

CONCEPTUAL QUESTIONS

6.1 Blackbody Radiation

1. Which surface has a higher temperature – the surface of a yellow star or that of a red star?

2. Describe what you would see when looking at a body whose temperature is increased from 1000 K to 1,000,000 K.

3. Explain the color changes in a hot body as its temperature is increased.

4. Speculate as to why UV light causes sunburn, whereas visible light does not.

5. Two cavity radiators are constructed with walls made of different metals. At the same temperature, how would their radiation spectra differ?

6. Discuss why some bodies appear black, other bodies appear red, and still other bodies appear white.

7. If everything radiates electromagnetic energy, why can we not see objects at room temperature in a dark room?

8. How much does the power radiated by a blackbody increase when its temperature (in K) is tripled?

6.2 Photoelectric Effect

9. For the same monochromatic light source, would the photoelectric effect occur for all metals?

10. In the interpretation of the photoelectric effect, how is it known that an electron does not absorb more than one photon?

11. Explain how you can determine the work function from a plot of the stopping potential versus the frequency of the incident radiation in a photoelectric effect experiment. Can you determine the value of Planck's constant from this plot?

12. Suppose that in the photoelectric-effect experiment we make a plot of the detected current versus the applied potential difference. What information do we obtain from such a plot? Can we determine from it the value of Planck's constant? Can we determine the work function of the metal?

13. Speculate how increasing the temperature of a photoelectrode affects the outcomes of the photoelectric effect experiment.

14. Which aspects of the photoelectric effect cannot be explained by classical physics?

15. Is the photoelectric effect a consequence of the wave character of radiation or is it a consequence of the particle character of radiation? Explain briefly.

16. The metals sodium, iron, and molybdenum have work functions 2.5 eV, 3.9 eV, and 4.2 eV, respectively. Which of these metals will emit photoelectrons when illuminated with 400 nm light?

6.3 The Compton Effect

17. Discuss any similarities and differences between the photoelectric and the Compton effects.

18. Which has a greater momentum: an UV photon or an IR photon?

19. Does changing the intensity of a monochromatic light beam affect the momentum of the individual photons in the beam? Does such a change affect the net momentum of the beam?

20. Can the Compton effect occur with visible light? If so, will it be detectable?

21. Is it possible in the Compton experiment to observe scattered X-rays that have a shorter wavelength than the incident X-ray radiation?

22. Show that the Compton wavelength has the dimension of length.

23. At what scattering angle is the wavelength shift in the Compton effect equal to the Compton wavelength?

6.4 Bohr's Model of the Hydrogen Atom

24. Explain why the patterns of bright emission spectral lines have an identical spectral position to the pattern of dark absorption spectral lines for a given gaseous element.

25. Do the various spectral lines of the hydrogen atom overlap?

26. The Balmer series for hydrogen was discovered before either the Lyman or the Paschen series. Why?

27. When the absorption spectrum of hydrogen at room temperature is analyzed, absorption lines for the Lyman series are found, but none are found for the Balmer series. What does this tell us about the energy state of most hydrogen atoms at room temperature?

28. Hydrogen accounts for about 75% by mass of the matter at the surfaces of most stars. However, the absorption lines of hydrogen are strongest (of highest intensity) in the spectra of stars with a surface temperature of about 9000 K. They are weaker in the sun spectrum and are essentially nonexistent in very hot (temperatures above 25,000 K) or rather cool (temperatures below 3500 K) stars. Speculate as to why surface temperature affects the hydrogen absorption lines that we observe.

29. Discuss the similarities and differences between

Thomson's model of the hydrogen atom and Bohr's model of the hydrogen atom.

30. Discuss the way in which Thomson's model is nonphysical. Support your argument with experimental evidence.

31. If, in a hydrogen atom, an electron moves to an orbit with a larger radius, does the energy of the hydrogen atom increase or decrease?

32. How is the energy conserved when an atom makes a transition from a higher to a lower energy state?

33. Suppose an electron in a hydrogen atom makes a transition from the ($n+1$)th orbit to the nth orbit. Is the wavelength of the emitted photon longer for larger values of n, or for smaller values of n?

34. Discuss why the allowed energies of the hydrogen atom are negative.

35. Can a hydrogen atom absorb a photon whose energy is greater than 13.6 eV?

36. Why can you see through glass but not through wood?

37. Do gravitational forces have a significant effect on atomic energy levels?

38. Show that Planck's constant has the dimensions of angular momentum.

6.5 De Broglie's Matter Waves

39. Which type of radiation is most suitable for the observation of diffraction patterns on crystalline solids; radio waves, visible light, or X-rays? Explain.

40. Speculate as to how the diffraction patterns of a typical crystal would be affected if γ-rays were used instead of X-rays.

41. If an electron and a proton are traveling at the same speed, which one has the shorter de Broglie wavelength?

42. If a particle is accelerating, how does this affect its de Broglie wavelength?

43. Why is the wave-like nature of matter not observed every day for macroscopic objects?

44. What is the wavelength of a neutron at rest? Explain.

45. Why does the setup of Davisson–Germer experiment need to be enclosed in a vacuum chamber? Discuss what result you expect when the chamber is not evacuated.

6.6 Wave-Particle Duality

46. Give an example of an experiment in which light behaves as waves. Give an example of an experiment in which light behaves as a stream of photons.

47. Discuss: How does the interference of water waves differ from the interference of electrons? How are they analogous?

48. Give at least one argument in support of the matter-wave hypothesis.

49. Give at least one argument in support of the particle-nature of radiation.

50. Explain the importance of the Young double-slit experiment.

51. Does the Heisenberg uncertainty principle allow a particle to be at rest in a designated region in space?

52. Can the de Broglie wavelength of a particle be known exactly?

53. Do the photons of red light produce better resolution in a microscope than blue light photons? Explain.

54. Discuss the main difference between an SEM and a TEM.

PROBLEMS

6.1 Blackbody Radiation

55. A 200-W heater emits a 1.5-µm radiation. (a) What value of the energy quantum does it emit? (b) Assuming that the specific heat of a 4.0-kg body is $0.83\,\text{kcal/kg}\cdot\text{K}$, how many of these photons must be absorbed by the body to increase its temperature by 2 K? (c) How long does the heating process in (b) take, assuming that all radiation emitted by the heater gets absorbed by the body?

56. A 900-W microwave generator in an oven generates energy quanta of frequency 2560 MHz. (a) How many energy quanta does it emit per second? (b) How many energy quanta must be absorbed by a pasta dish placed in the radiation cavity to increase its temperature by 45.0 K? Assume that the dish has a mass of 0.5 kg and that its specific heat is $0.9\,\text{kcal/kg} \cdot \text{K}$. (c) Assume that all energy quanta emitted by the generator are absorbed by the pasta dish. How long must we wait until the dish in (b) is ready?

57. (a) For what temperature is the peak of blackbody radiation spectrum at 400 nm? (b) If the temperature of a blackbody is 800 K, at what wavelength does it radiate the most energy?

58. The tungsten elements of incandescent light bulbs operate at 3200 K. At what frequency does the filament radiate maximum energy?

59. Interstellar space is filled with radiation of wavelength $970\,\mu\text{m}$. This radiation is considered to be a remnant of the "big bang." What is the corresponding blackbody temperature of this radiation?

60. The radiant energy from the sun reaches its maximum at a wavelength of about 500.0 nm. What is the approximate temperature of the sun's surface?

6.2 Photoelectric Effect

61. A photon has energy 20 keV. What are its frequency and wavelength?

62. The wavelengths of visible light range from approximately 400 to 750 nm. What is the corresponding range of photon energies for visible light?

63. What is the longest wavelength of radiation that can eject a photoelectron from silver? Is it in the visible range?

64. What is the longest wavelength of radiation that can eject a photoelectron from potassium, given the work function of potassium 2.24 eV? Is it in the visible range?

65. Estimate the binding energy of electrons in magnesium, given that the wavelength of 337 nm is the longest wavelength that a photon may have to eject a photoelectron from magnesium photoelectrode.

66. The work function for potassium is 2.26 eV. What is the cutoff frequency when this metal is used as photoelectrode? What is the stopping potential when for the emitted electrons when this photoelectrode is exposed to radiation of frequency 1200 THz?

67. Estimate the work function of aluminum, given that the wavelength of 304 nm is the longest wavelength that a photon may have to eject a photoelectron from aluminum photoelectrode.

68. What is the maximum kinetic energy of photoelectrons ejected from sodium by the incident radiation of wavelength 450 nm?

69. A 120-nm UV radiation illuminates a gold-plated electrode. What is the maximum kinetic energy of the ejected photoelectrons?

70. A 400-nm violet light ejects photoelectrons with a maximum kinetic energy of 0.860 eV from sodium photoelectrode. What is the work function of sodium?

71. A 600-nm light falls on a photoelectric surface and electrons with the maximum kinetic energy of 0.17 eV are emitted. Determine (a) the work function and (b) the cutoff frequency of the surface. (c) What is the stopping potential when the surface is illuminated with light of wavelength 400 nm?

72. The cutoff wavelength for the emission of photoelectrons from a particular surface is 500 nm. Find the maximum kinetic energy of the ejected photoelectrons when the surface is illuminated with light of wavelength 600 nm.

73. Find the wavelength of radiation that can eject 2.00-eV electrons from calcium electrode. The work function for calcium is 2.71 eV. In what range is this radiation?

74. Find the wavelength of radiation that can eject 0.10-eV electrons from potassium electrode. The work function for potassium is 2.24 eV. In what range is this radiation?

75. Find the maximum velocity of photoelectrons ejected by an 80-nm radiation, if the work function of photoelectrode is 4.73 eV.

6.3 The Compton Effect

76. What is the momentum of a 589-nm yellow photon?

77. What is the momentum of a 4-cm microwave photon?

78. In a beam of white light (wavelengths from 400 to 750 nm), what range of momentum can the photons have?

79. What is the energy of a photon whose momentum is $3.0 \times 10^{-24}\,\text{kg} \cdot \text{m/s}$?

80. What is the wavelength of (a) a 12-keV X-ray photon;

(b) a 2.0-MeV γ -ray photon?

81. Find the momentum and energy of a 1.0-Å photon.

82. Find the wavelength and energy of a photon with momentum $5.00 \times 10^{-29}\,\text{kg} \cdot \text{m/s}$.

83. A γ -ray photon has a momentum of $8.00 \times 10^{-21}\,\text{kg} \cdot \text{m/s}$. Find its wavelength and energy.

84. (a) Calculate the momentum of a $2.5\text{-}\mu\text{m}$ photon. (b) Find the velocity of an electron with the same momentum. (c) What is the kinetic energy of the electron, and how does it compare to that of the photon?

85. Show that $p = h/\lambda$ and $E_f = hf$ are consistent with the relativistic formula $E^2 = p^2c^2 + m_0^2c^2$.

86. Show that the energy *E* in eV of a photon is given by $E = 1.241 \times 10^{-6}\,\text{eV} \cdot \text{m}/\lambda$, where λ is its wavelength in meters.

87. For collisions with free electrons, compare the Compton shift of a photon scattered as an angle of $30°$ to that of a photon scattered at $45°$.

88. X-rays of wavelength 12.5 pm are scattered from a block of carbon. What are the wavelengths of photons scattered at (a) $30°$; (b) $90°$; and, (c) $180°$?

6.4 Bohr’s Model of the Hydrogen Atom

89. Calculate the wavelength of the first line in the Lyman series and show that this line lies in the ultraviolet part of the spectrum.

90. Calculate the wavelength of the fifth line in the Lyman series and show that this line lies in the ultraviolet part of the spectrum.

91. Calculate the energy changes corresponding to the transitions of the hydrogen atom: (a) from $n = 3$ to $n = 4$; (b) from $n = 2$ to $n = 1$; and (c) from $n = 3$ to $n = \infty$.

92. Determine the wavelength of the third Balmer line (transition from $n = 5$ to $n = 2$).

93. What is the frequency of the photon absorbed when the hydrogen atom makes the transition from the ground state to the $n = 4$ state?

94. When a hydrogen atom is in its ground state, what are the shortest and longest wavelengths of the photons it can absorb without being ionized?

95. When a hydrogen atom is in its third excided state, what are the shortest and longest wavelengths of the photons it can emit?

96. What is the longest wavelength that light can have if it is to be capable of ionizing the hydrogen atom in its ground state?

97. For an electron in a hydrogen atom in the $n = 2$ state, compute: (a) the angular momentum; (b) the kinetic energy; (c) the potential energy; and (d) the total energy.

98. Find the ionization energy of a hydrogen atom in the fourth energy state.

99. It has been measured that it required 0.850 eV to remove an electron from the hydrogen atom. In what state was the atom before the ionization happened?

100. What is the radius of a hydrogen atom when the electron is in the first excited state?

101. Find the shortest wavelength in the Balmer series. In what part of the spectrum does this line lie?

102. Show that the entire Paschen series lies in the infrared part of the spectrum.

103. Do the Balmer series and the Lyman series overlap? Why? Why not? (Hint: calculate the shortest Balmer line and the longest Lyman line.)

104. (a) Which line in the Balmer series is the first one in the UV part of the spectrum? (b) How many Balmer lines lie in the visible part of the spectrum? (c) How many Balmer lines lie in the UV?

105. A $4.653\text{-}\mu\text{m}$ emission line of atomic hydrogen corresponds to transition between the states $n_f = 5$ and n_i. Find n_i.

6.5 De Broglie’s Matter Waves

106. At what velocity will an electron have a wavelength of 1.00 m?

107. What is the de Broglie wavelength of an electron travelling at a speed of $5.0 \times 10^6 \text{ m/s}$?

108. What is the de Broglie wavelength of an electron that is accelerated from rest through a potential difference of 20 keV?

109. What is the de Broglie wavelength of a proton whose kinetic energy is 2.0 MeV? 10.0 MeV?

110. What is the de Broglie wavelength of a 10-kg football player running at a speed of 8.0 m/s?

111. (a) What is the energy of an electron whose de Broglie wavelength is that of a photon of yellow light with wavelength 590 nm? (b) What is the de Broglie wavelength of an electron whose energy is that of the photon of yellow light?

112. The de Broglie wavelength of a neutron is 0.01 nm. What is the speed and energy of this neutron?

113. What is the wavelength of an electron that is moving at a 3% of the speed of light?

114. At what velocity does a proton have a 6.0-fm wavelength (about the size of a nucleus)? Give your answer in units of *c*.

115. What is the velocity of a 0.400-kg billiard ball if its wavelength is 7.50 fm?

116. Find the wavelength of a proton that is moving at 1.00% of the speed of light (when $\beta = 0.01$).

6.6 Wave-Particle Duality

117. An AM radio transmitter radiates 500 kW at a frequency of 760 kHz. How many photons per second does the emitter emit?

118. Find the Lorentz factor γ and de Broglie's wavelength for a 50-GeV electron in a particle accelerator.

119. Find the Lorentz factor γ and de Broglie's wavelength for a 1.0-TeV proton in a particle accelerator.

120. What is the kinetic energy of a 0.01-nm electron in a TEM?

121. If electron is to be diffracted significantly by a crystal, its wavelength must be about equal to the spacing, *d*, of crystalline planes. Assuming $d = 0.250 \text{ nm}$, estimate the potential difference through which an electron must be accelerated from rest if it is to be diffracted by these planes.

122. X-rays form ionizing radiation that is dangerous to living tissue and undetectable to the human eye. Suppose that a student researcher working in an X-ray diffraction laboratory is accidentally exposed to a fatal dose of radiation. Calculate the temperature increase of the researcher under the following conditions: the energy of X-ray photons is 200 keV and the researcher absorbs 4×10^{13} photons per each kilogram of body weight during the exposure. Assume that the specific heat of the student's body is $0.83 \text{kcal/kg} \cdot \text{K}$.

123. Solar wind (radiation) that is incident on the top of Earth's atmosphere has an average intensity of 1.3kW/m^2. Suppose that you are building a solar sail that is to propel a small toy spaceship with a mass of 0.1 kg in the space between the International Space Station and the moon. The sail is made from a very light material, which perfectly reflects the incident radiation. To assess whether such a project is feasible, answer the following questions, assuming that radiation photons are incident only in normal direction to the sail reflecting surface. (a) What is the radiation pressure (force per m^2) of the radiation falling on the mirror-like sail? (b) Given the radiation pressure computed in (a), what will be the acceleration of the spaceship when the sail has of an area of 10.0m^2 ? (c) Given the acceleration estimate in (b), how fast will the spaceship be moving after 24 hours when it starts from rest?

124. Treat the human body as a blackbody and determine the percentage increase in the total power of its radiation when its temperature increases from 98.6 ° F to 103 ° F.

125. Show that Wien's displacement law results from Planck's radiation law. (*Hint:* substitute $x = hc/\lambda kT$ and write Planck's law in the form $I(x, T) = Ax^5/(e^x - 1)$, where $A = 2\pi(kT)^5/(h^4 c^3)$. Now, for fixed *T*, find the position of the maximum in *I*(*x*,*T*) by solving for *x* in the equation $dI(x, T)/dx = 0.$)

126. Show that Stefan's law results from Planck's radiation law. *Hint:* To compute the total power of blackbody radiation emitted across the entire spectrum of wavelengths at a given temperature, integrate Planck's law over the entire spectrum $P(T) = \int_0^\infty I(\lambda, T) d\lambda$. Use the substitution $x = hc/\lambda kT$ and the tabulated value of the integral $\int_0^\infty dx x^3/(e^x - 1) = \pi^4/15$.

ADDITIONAL PROBLEMS

127. Determine the power intensity of radiation per unit wavelength emitted at a wavelength of 500.0 nm by a blackbody at a temperature of 10,000 K.

128. The HCl molecule oscillates at a frequency of 87.0 THz. What is the difference (in eV) between its adjacent energy levels?

129. A quantum mechanical oscillator vibrates at a frequency of 250.0 THz. What is the minimum energy of radiation it can emit?

130. In about 5 billion years, the sun will evolve to a red giant. Assume that its surface temperature will decrease to about half its present value of 6000 K, while its present radius of $7.0 \times 10^8\,\text{m}$ will increase to $1.5 \times 10^{11}\,\text{m}$ (which is the current Earth-sun distance). Calculate the ratio of the total power emitted by the sun in its red giant stage to its present power.

131. A sodium lamp emits 2.0 W of radiant energy, most of which has a wavelength of about 589 nm. Estimate the number of photons emitted per second by the lamp.

132. Photoelectrons are ejected from a photoelectrode and are detected at a distance of 2.50 cm away from the photoelectrode. The work function of the photoelectrode is 2.71 eV and the incident radiation has a wavelength of 420 nm. How long does it take a photoelectron to travel to the detector?

133. If the work function of a metal is 3.2 eV, what is the maximum wavelength that a photon can have to eject a photoelectron from this metal surface?

134. The work function of a photoelectric surface is 2.00 eV. What is the maximum speed of the photoelectrons emitted from this surface when a 450-nm light falls on it?

135. A 400-nm laser beam is projected onto a calcium electrode. The power of the laser beam is 2.00 mW and the work function of calcium is 2.31 eV. (a) How many photoelectrons per second are ejected? (b) What net power is carried away by photoelectrons?

136. (a) Calculate the number of photoelectrons per second that are ejected from a 1.00-mm^2 area of sodium metal by a 500-nm radiation with intensity 1.30kW/m^2 (the intensity of sunlight above Earth's atmosphere). (b) Given the work function of the metal as 2.28 eV, what power is carried away by these photoelectrons?

137. A laser with a power output of 2.00 mW at a 400-nm wavelength is used to project a beam of light onto a calcium photoelectrode. (a) How many photoelectrons leave the calcium surface per second? (b) What power is carried away by ejected photoelectrons, given that the work function of calcium is 2.31 eV? (c) Calculate the photocurrent. (d) If the photoelectrode suddenly becomes electrically insulated and the setup of two electrodes in the circuit suddenly starts to act like a 2.00-pF capacitor, how long will current flow before the capacitor voltage stops it?

138. The work function for barium is 2.48 eV. Find the maximum kinetic energy of the ejected photoelectrons when the barium surface is illuminated with: (a) radiation emitted by a 100-kW radio station broadcasting at 800 kHz; (b) a 633-nm laser light emitted from a powerful He-Ne laser; and (c) a 434-nm blue light emitted by a small hydrogen gas discharge tube.

139. (a) Calculate the wavelength of a photon that has the same momentum as a proton moving with 1% of the speed of light in a vacuum. (b) What is the energy of this photon in MeV? (c) What is the kinetic energy of the proton in MeV?

140. (a) Find the momentum of a 100-keV X-ray photon. (b) Find the velocity of a neutron with the same momentum. (c) What is the neutron's kinetic energy in eV?

141. The momentum of light, as it is for particles, is exactly reversed when a photon is reflected straight back from a mirror, assuming negligible recoil of the mirror. The change in momentum is twice the photon's incident momentum, as it is for the particles. Suppose that a beam of light has an intensity 1.0kW/m^2 and falls on a -2.0-m^2 area of a mirror and reflects from it. (a) Calculate the energy reflected in 1.00 s. (b) What is the momentum imparted to the mirror? (c) Use Newton's second law to find the force on the mirror. (d) Does the assumption of no-recoil for the mirror seem reasonable?

142. A photon of energy 5.0 keV collides with a stationary electron and is scattered at an angle of $60°$. What is the energy acquired by the electron in the collision?

143. A 0.75-nm photon is scattered by a stationary electron. The speed of the electron's recoil is $1.5 \times 10^6\,\text{m/s}$. (a) Find the wavelength shift of the photon. (b) Find the scattering angle of the photon.

144. Find the maximum change in X-ray wavelength that can occur due to Compton scattering. Does this change depend on the wavelength of the incident beam?

145. A photon of wavelength 700 nm is incident on a hydrogen atom. When this photon is absorbed, the atom becomes ionized. What is the lowest possible orbit that the electron could have occupied before being ionized?

146. What is the maximum kinetic energy of an electron such that a collision between the electron and a stationary hydrogen atom in its ground state is definitely elastic?

147. Singly ionized atomic helium He^{+1} is a hydrogen-like ion. (a) What is its ground-state radius? (b) Calculate the energies of its four lowest energy states. (c) Repeat the calculations for the Li^{2+} ion.

148. A triply ionized atom of beryllium Be^{3+} is a hydrogen-like ion. When Be^{3+} is in one of its excited states, its radius in this *n*th state is exactly the same as the radius of the first Bohr orbit of hydrogen. Find *n* and compute the ionization energy for this state of Be^{3+}.

149. In extreme-temperature environments, such as those existing in a solar corona, atoms may be ionized by undergoing collisions with other atoms. One example of such ionization in the solar corona is the presence of C^{5+} ions, detected in the Fraunhofer spectrum. (a) By what factor do the energies of the C^{5+} ion scale compare to the energy spectrum of a hydrogen atom? (b) What is the wavelength of the first line in the Paschen series of C^{5+} ? (c) In what part of the spectrum are these lines located?

150. (a) Calculate the ionization energy for He^{+}. (b) What is the minimum frequency of a photon capable of ionizing He^{+} ?

151. Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. Find the wavelength of such an ultracold neutron and its kinetic energy.

152. Find the velocity and kinetic energy of a 6.0-fm neutron. (Rest mass energy of neutron is $E_0 = 940\,\text{MeV}$.)

153. The spacing between crystalline planes in the NaCl crystal is 0.281 nm, as determined by X-ray diffraction with X-rays of wavelength 0.170 nm. What is the energy of neutrons in the neutron beam that produces diffraction peaks at the same locations as the peaks obtained with the X-rays?

154. What is the wavelength of an electron accelerated from rest in a 30.0-kV potential difference?

155. Calculate the velocity of a $1.0\text{-}\mu\text{m}$ electron and a potential difference used to accelerate it from rest to this velocity.

156. In a supercollider at CERN, protons are accelerated to velocities of 0.25*c*. What are their wavelengths at this speed? What are their kinetic energies? If a beam of protons were to gain its kinetic energy in only one pass through a potential difference, how high would this potential difference have to be? (Rest mass energy of a proton is $E_0 = 938\,\text{MeV}$).

157. Find the de Broglie wavelength of an electron accelerated from rest in an X-ray tube in the potential difference of 100 keV. (Rest mass energy of an electron is $E_0 = 511\,\text{keV}$.)

158. The cutoff wavelength for the emission of photoelectrons from a particular surface is 500 nm. Find the maximum kinetic energy of the ejected photoelectrons when the surface is illuminated with light of wavelength 450 nm.

159. Compare the wavelength shift of a photon scattered by a free electron to that of a photon scattered at the same angle by a free proton.

160. The spectrometer used to measure the wavelengths of the scattered X-rays in the Compton experiment is accurate to $5.0 \times 10^{-4}\,\text{nm}$. What is the minimum scattering angle for which the X-rays interacting with the free electrons can be distinguished from those interacting with the atoms?

161. Consider a hydrogen-like ion where an electron is orbiting a nucleus that has charge $q = +Ze$. Derive the formulas for the energy E_n of the electron in *n*th orbit and the orbital radius r_n.

162. Assume that a hydrogen atom exists in the $n = 2$ excited state for $10^{-8}\,\text{s}$ before decaying to the ground state. How many times does the electron orbit the proton nucleus during this time? How long does it take Earth to orbit the sun this many times?

163. An atom can be formed when a negative muon is captured by a proton. The muon has the same charge as the electron and a mass 207 times that of the electron. Calculate the frequency of the photon emitted when this atom makes the transition from $n = 2$ to the $n = 1$ state. Assume that the muon is orbiting a stationary proton.